What Mathematics Should *All* College Students Know?

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William L. Briggs
University of Colorado at Denver
wbriggs@math.cudenver.edu

Marie and Alex just paid $250,000 for a house. They made a down payment of $50,000 and assumed a 30-year $200,000 mortgage with a fixed annual interest rate of 7.50%. The house will serve as a residence for several years, but Marie and Alex also view it as an investment, as property values in the neighborhood are projected to increase at a rate of 5% per year in the near future. Suppose the couple sells the house after eight years. Neglecting income tax deductions, do they come out ahead on their investment?[1]

This question doesn't sound like one encountered in most mathematics courses. First, while it is a problem in words, it is not the dreaded word problem that many of us remember from high school mathematics courses. Second, the problem is relevant and immediate; it is unlikely that a student would respond to this question with the familiar "what does this have to do with my life." Third, while the solution involves fairly elementary mathematics, it is a multi-step process that requires organizing several pieces of information. Fourth, the solution requires some understanding of home mortgages and appreciation of property values; these topics are not considered mathematics, but they represent *applications* of mathematics. Finally, the problem invites discussion and extensions: What assumptions were made in arriving at an answer? How would the answer change if Marie and Alex are in a 28% tax bracket and income tax deductions are considered? How would the answer change if the interest rate were 8.00% or the appreciation rate were 4% per year? What is the minimum time that Marie and Alex must live in the house before they break even?

For the past twenty years, mathematics educators have been in the midst of a reform. Driven by a series of influential reports concerning the deteriorating state of mathematics education [1, 2, 3, 4], many mathematics teachers have changed the way they teach. For example, they appreciate the diversity of learning styles in a single classroom; they use computers and calculators in wise ways to extend, rather than replace, students' quantitative skills; and they balance the practical uses of mathematics with more abstract topics. It may be too soon to assess the ultimate outcome of these reform efforts; but many educators believe that they will have a lasting impact on the teaching of mathematics.

However, this reform has been directed largely at students who are in the "calculus pipeline," students taking calculus or precalculus courses. These so-called SMET (science, mathematics, engineering, and technology) students comprise a relatively small fraction of all college and university students who take mathematics courses. Easily half of all students who take mathematics courses are *not* calculus-bound and do so only to satisfy a core curriculum or general education requirement. These non-SMET students are primarily liberal arts students and they have not benefited greatly from the recent reforms. A pressing question remains: What sort of mathematics course should we offer our liberal arts students? Or, what mathematics should *all* college students know?

[1] After 8 years, they have paid approximately $120,000 in interest and their property has appreciated by $119,360. So 8 years is very close to the break-even point.

Several observations bear on the teaching of liberal arts mathematics and explain why it is so challenging. First, teaching such courses is not a high priority of many mathematics departments and it is not the concern of many full-time faculty members. As a result, the design and teaching of these courses is often neglected.

Second, students who take liberal arts mathematics courses often are victims of previous mathematics courses and instructors. Not surprisingly, they harbor genuine fears of mathematics, have lost confidence in their quantitative skills, and have little belief that mathematics might be of use in their future. It is unfortunate that because of poor advising or lack of alternatives, many of these students mistakenly end up in the calculus pipeline, taking a college algebra course. The outcome is often catastrophic.

Third, because of the second observation, providing liberal arts students with a worthwhile experience in their last mathematics course requires overcoming significant psychological obstacles. It *cannot* be done by subjecting students to more of the same experiences they have had in previous mathematics courses. It *can* be done by demonstrating the breadth and utility of mathematics with compelling examples of how it affects students' lives in immediate ways.

Finally, most mathematics educators have a shared understanding of the content of an algebra course or a calculus course. By contrast, there is no common agreement, at the moment, about the content and expectations of a liberal arts mathematics course. Indeed, transferring such courses between institutions is often difficult.

The task is further complicated by the *algebra dilemma*, one side of which is the belief that a minimally educated person must be proficient with the abstractions and manipulations of algebra. Risking analogies with other disciplines, this claim might be compared to the belief that a student must be able to identify a diminished seventh in order to have a lifelong appreciation of music, or that Sartre must be read in French in order to understand his philosophy. For calculus-bound students, there is no question that algebra is a gatekeeper and its thorough mastery is essential. For a more general audience, such as liberal arts students, selected algebraic skills are important; but there are other, equally vital topics and skills.

For those of us who have spent much of a lifetime studying, using, and teaching mathematics, it is difficult to concede that when it comes to algebra, less could be better. However, this concession is an important key to designing a successful liberal arts mathematics courses. As the story of Marie and Alex, and the examples below illustrate, it is possible to identify a wealth of fascinating mathematics and relevant applications that do not rely on extensive algebra and symbolic manipulations. A useful principle is to avoid doing algebra when there is no ulterior purpose and to let the applications determine the necessary mathematics. If an important application requires some algebra, then it is always possible to isolate that particular skill and focus on its mastery, without introducing unnecessary generalizations.

Achieving a reasonable balance with respect to the role of algebra leaves room in a one-semester course for other truly essential topics. These topics should prepare students for careers and lives that will be filled with quantitative information and decisions. For example,

- students must possess strong critical and logical thinking skills, so they can navigate the media and be informed citizens;
- they should have a strong number sense and be proficient at estimation, unit conversions, and the uses of percentages;
- they should be able to read a statistical study – or at least a summary – and evaluate it critically;
- they should possess the mathematical tools needed to make basic financial decisions; and

- they should understand exponential growth and know that it governs everything from populations and prices to tumors and drugs in the blood.

Any remaining time (or a second semester) can be filled with a wealth of breadth topics, such as risk analysis, voting, apportionment, mathematics and the arts, and graph theory, to name only a few.

The consequence of being selective about algebra and including a variety of practical topics is not a watered down mathematics course. The tradeoff is that mathematics becomes part of a larger set of skills, often called *quantitative literacy* or *numeracy*, which involves critical thinking, problem formulation, and written and oral communication. The quantitative reasoning approach allows students to see mathematics in a larger interdisciplinary setting that provides new problem-solving and decision-making powers. It presents mathematics *in context*, as a discipline that is connected to the world around them and essential to an understanding of that world. It also provides students with a much broader survey of mathematics and statistics than afforded by other courses.

As the story of Marie and Alex demonstrates, typical problems in an effective quantitative reasoning course involve the application of relatively elementary mathematics to practical situations. They are often open-ended problems that disabuse students of the belief that answers to mathematics problems are unique and always given in the back of the book. They may involve the use of library or Internet resources for background information. And they have the goal of strengthening students' problem solving confidence and communications skills.

We close with an attempt to answer the title question, what mathematics should all college students know? The following proficiency questions reflect core skills and concepts that should be familiar to a quantitatively literate college student. While the specific context of these questions (including the Marie and Alex story) may vary, they involve no mathematics or prerequisite material beyond what might be covered in a typical quantitative literacy course.

Quantitative Literacy Proficiency Test

1. The following ballot initiative appeared before Colorado voters in 1992:

Shall there be an amendment to the Colorado constitution to prohibit the state of Colorado and any of its political subdivisions from adopting or enforcing any law or policy which provides that homosexual, lesbian, or bisexual orientation, conduct, or relationships constitutes or entitles a person to claim any minority or protected status, quota preferences, or discrimination?

What does a *yes* vote mean?[2]

2. Suppose that the United States government decided to institute a national lottery, the proceeds of which would be used to retire the federal debt (which can be taken to be $7 trillion). Assume that the lottery could raise $50 million each week after expenses and prizes. How long would it take to pay off the federal debt assuming that the budget is exactly balanced every year in the future?[3]

3. You've been charging your school expenses to a credit card, and have built up a balance of $5,000. Your credit card charges an annual interest rate of 18%. Assume you charge nothing more to your credit card.

a. At this rate, what is your monthly payment for interest only?

b. Suppose the credit card requires that you make minimum monthly payments of $70. With minimum payments, how long will it take you to pay off the balance?[4]

4. At the local Video Station, you pay a total of $15.50 for a DVD, after taxes. Assuming a

[2] A *yes* vote is a vote *against* protected status for homosexual, lesbian, or bisexual people.
[3] It would take about 2700 years to pay of the debt.
[4] (a) Monthly payments are $75. (b) At a rate of $70 per month, the balance will never get paid off.

local sales tax rate of 7.5%, what is the retail (before-tax) price of the DVD?[5]

5. You have a fair coin, which means the chance of getting a head on a single toss is 1/2. Suppose you toss the coin 10 times and get 10 tails in a row. Is the probability of getting a head on your next toss more than, less than, or equal to 1/2? Explain.[6]

6. Last year, it was bad news: the value of your investments plunged by 50%. This year was much better: your investments increased in value by 75%. Over the two-year period, have you gained or lost? Explain.[7]

7. An election features three candidates: Smith, Jones, and Webb. Among the actual voters, Webb is by far the most disliked candidate; in fact, 60% of the voters oppose his election. Is it still possible for Webb to win? Explain.[8]

8. A high school tests all athletes for drug use, using a drug test that is 98% accurate. That is, it correctly gives a positive result for 98% of the drug users who are tested, and it correctly gives a negative result for 98% of the nonusers who are tested. Suppose that 1,000 athletes take the test, and 50 of these athletes are actually using drugs. What percentage of the positive tests are false positives (nonusers who test positive)?[9]

9. The world population is currently about 6 billion and increasing at a rate of 1.3% per year. If this rate remains constant, how long will it take the population to double in size? At this rate, estimate the world population in the year 2100.[10]

10. Suppose you are visiting an Italian market and see tomatoes priced at 3.20 Euros per kilogram. Assume that 1 kilogram = 2.2 pounds and that the current exchange rate is $1 = 0.9 Euros. What is the price of the tomatoes in dollars per pound?[11]

11. You purchase 10 tickets for a lottery in which the probability of winning any prize on a single ticket is 1 in 10.

a. What is the probability that you'll have *at least one* winning ticket among the 10 tickets?

b. What is the probability that all 10 tickets are losers?[12]

12. The figure below shows the increase in tuition at public and private colleges and universities between 1987 and 1995. The increase in the consumer price index (CPI) over the same period is also shown.

a. In what year between 1987 and 1995 was the tuition at public schools the greatest? Explain.

b. Which increased more between 1987 and 1995, tuition at private schools or the cost of living? Explain.

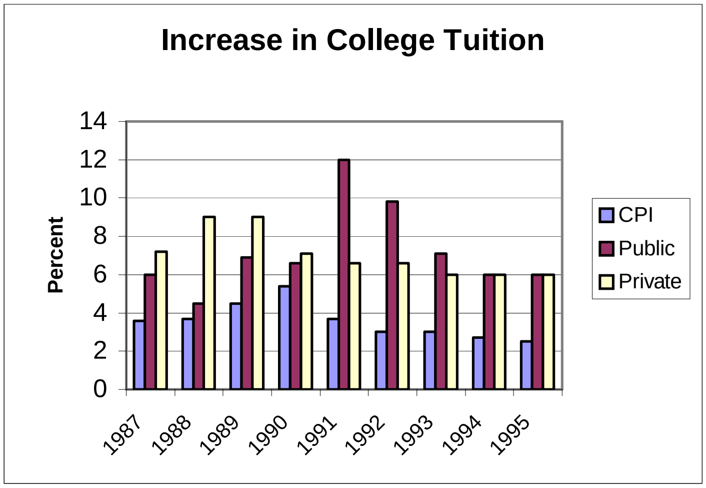
c. Have there been any years in which tuition decreased at either public or private colleges? Explain.[13]

The development of effective mathematics courses for liberal arts students is happening, albeit on a slow time scale. However, the "client departments" that comprise the liberal arts can hasten the pace. These departments must insist that liberal arts mathematics courses be given the attention lavished on more advanced mathematics courses. They must express to the mathematics faculty their expectations of a valuable mathematical experience for their students. In the end, the development of these courses should be a collaborative and interdisciplinary effort, in

[5] The pre-tax price is $14.42.
[6] The probability of a head on the next toss is still ½.
[7] The value of the investments stand a $0.5 \times 1.75 = 0.875$ of their initial value.
[8] Webb could get 40% of the votes, while Smith and Jones get 30%of the vote each.
[9] Almost 28% of the positive tests are false positives.
[10] The doubling time is roughly 55 years. At this rate, the population in 2100 will be about 21 billion.

[11] The price is $1.62 per pound.
[12] (a) The probability of at least one winner is 0.65. (b) The probability of 10 losers is 0.35.
[13] (a) The tuition at public schools increased every year, so it was greatest in 1995. (b) Tuition at private schools increased more between 1987 and 1995. (c) Tuition increased in every year.

which mathematics departments either participate or take the lead. The result will be a rising tide of quantitative skills among all students, which is a worthy cause to be sure.



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