Master Syllabus for MATH 1401 (Calculus I)
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Preface

- This syllabus is aligned with *Calculus: Early Transcendentals,* 1e, Briggs & Cochran [Addison Wesley, 2010].

(#1) Idea of Limits

(a) Average velocity; Slope of secant line segment.
(b) Instantaneous velocity; Slope of tangent line.
[By Euclidean geometry, tangent lines are only defined on circles. How do we define tangent lines for other curves?]

(#2) Definition of Limits

(a) Preliminary definition for

$$\lim_{x \to a} f(x) = L.$$

(b) Finding limits from a graph.
(c) Finding limits from a table (numerical).
(d) Relationship between one-sided and two-sided limits.
(e) Jump discontinuities.
(f) Situations where no limit exists. Example: $f(x) = \cos(1/x)$ as $x \to 0^+$.

(#3) Computing Limits

(a) Limits of linear functions; Direct substitution for continuous functions.
(b) Limit laws.
(c) Limits of rational functions; Cancellation rules.
(d) Limits which require clever techniques. Example: algebraic conjugate.
(e) Squeeze Theorem. Show that

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1.$$

(#4) Infinite Limits

(a) Extended real numbers, $-\infty$ and ∞.
(b) Vertical asymptotes.

(#5) Limits at Infinity

(a) Horizontal asymptotes
(b) Left and right end behaviors.
[This includes transcendental functions.]

(#6) Continuity

(a) Three conditions for continuity.
(b) General continuity rules (sum, product, powers, etc.).
(c) Continuity of composite functions and inverses.
(d) Continuity at end points (of an interval).
(e) Restate the Intermediate Value Theorem (from precalculus).

(#7) Introducing the Derivative

(a) Rate of change $\Leftrightarrow$ Slope of tangent line.
(b) At $x = a$, we have the slope of the tangent line:

$$m_{\tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

(c) In general, at any value of x, we have

$$f'(x) = \frac{df}{dx} = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

(d) Various notations for evaluating the derivative at a particular value of the independent variable.
(e) Comparing the graphs of $f'(x)$ with $f(x)$.
(f) Differentiability implies continuity. [The converse is not true.]

(#8) Rules for Differentiation

(a) Constant and Power Rules.
(b) Multiplicative constants. [Constant Multiple Rule.]
(c) Sum and Difference Rules.
(d) Exponential limit:

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1 \Rightarrow [e^x]' = e^x.$$

(e) Higher order derivatives.

(#9) Product & Quotient Rules

(a) Product Rule and its application to longer products like $[fgh]'$.
Interesting example: $e^{2x} = e^x e^x \Rightarrow \left[e^{2x}\right]' = 2e^{2x}$.
(b) Quotient Rule. [Give a mnemonic?]
It is okay to use Product Rule for everything, but advise students that often, the final answer must be expressed as a single fraction. Thus, the same or possibly more work will be incurred combining the two Product Rule terms together.
(c) Extending the Power Rule to negative integers.

(#10) Derivatives of Trigonometric Functions

(a) Show that
$$\lim_{x\to 0}\frac{\sin(x)}{x}=0\Rightarrow\lim_{x\to 0}\left(\frac{\cos(x)-1}{x}\right)=0.$$

(b) Derivatives of $\sin(x)$ and $\cos(x)$.

(c) Derivatives of $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$.
Mention cofunction rules in differentiation.

(d) Higher order derivatives (sine are cosine are cyclic).

(#11) Derivatives as Rates of Change

(a) Position functions; Average velocity.

(b) Instantaneous velocity; Speed function.

(c) Acceleration [s is sometimes arc length on a curve].

(d) Growth models.

(e) Average & marginal cost.

(#12) Chain Rule

(a) This really should be called the "Composition Rule".

(b) Identifying Outer [$y=f(u)$] and Inner [$u=g(x)$] functions in compositions:
$h(x)=f(g(x))$.

(c) General Power Rule.

(d) Calculating derivatives at a point.
[$f'(g(a))$ and $g'(a)$ must exist.]

(#13) Implicit Differentiation

(a) Application of Chain Rule:
$$\frac{d}{dx}[y^n]=ny^{n-1}y'.$$

(b) Power Rule for rational exponents.

(c) Procedures for finding higher order implicit derivatives.

(#14) Derivatives of Logarithmic and Exponential Functions

(a) Use implicit differentiation on $x=e^y$ to obtain $[\ln(x)]'=1/x$.

(b) Formulas for bases $b>0,\ b\neq 1$.

(c) General Power Rule for any integer n.

(d) Logarithmic differentiation.

(#15) Derivatives of Inverse Trigonometric Functions

(a) Use implicit differentiation to obtain $\left[\sin^{-1}(x)\right]' = \dfrac{1}{\sqrt{1-x^2}}$.

(b) Use cofunction rule to obtain $\left[\cos^{-1}(x)\right]'$.

(c) Derivatives of $\tan^{-1}(x)$ and $\sec^{-1}(x)$.

(d) Derivatives of inverse functions [general formula].

(#16) Related Rates

(a) More Chain Rule applications.
[Implicit differentiation with respect t.]

(b) Review procedures for solving related rates problems.

(#17) Maxima and Minima

(a) Absolute extrema.

(b) Extreme Value Theorem.

(c) Local extrema.

(d) Local Extreme Point Theorem (Leibniz); Critical points.

(e) Procedures for locating absolute extrema.

(#18) What Derivatives Tell Us

(a) Open intervals where $f(x)$ increases or decreases.

(b) Identifying local extrema; First Derivative Test

(c) Concavity and inflection points; Test for concavity.

(d) Second Derivative Test for local extrema.

(#19) Graphing Functions [Curve Sketching]

(a) Sketching guidelines.

(b) Sign charts. [Not approved for AP Calculus!]

(c) Functions with cusps and vertical tangent lines. Example: $f(x) = x^{2/3}$.

(#20) Optimization

(a) Objective functions.

(b) Review guidelines and procedures. Care must be taken when dealing with physical domains.

(c) Explore technology-based options.

(#21) Linear Approximation and Differentials

(a) Linear approximation to f at $x = a$; Errors to the approximation.

(b) Estimating changes with linear approximations.

(c) Differentials as change.

(#22) Mean Value Theorem

(a) Rolle's Theorem

(b) Mean Value Theorem and its consequences.

(c) Functions with equal derivatives differ by a constant.

(#23) L'Hôpital's Rule [Bernoulli's Theorem]

(a) Indeterminate forms: $0/0$ and ∞/∞.

(b) Rewriting forms: $(0 * \infty)$ and $(\infty - \infty)$.

(c) Indeterminate forms: $1^\infty,\ 0^0,\ \infty^0$.

(d) Growth rates of functions.

(e) Pitfalls in using l'Hôpital's.

(#24) Antiderivatives

(a) Indefinite integration.

(b) Power Rule.

(c) Constant multiple and sum rules.

(d) Indefinite integrals of trigonometric functions and inverses.

(e) Introduction to ordinary differential equations.

(f) Initial value problems for velocity and position functions.

(#25) Approximating Areas Under Curves

(a) Area under a velocity curve; Approximating displacement.

(b) Approximate areas by Riemann sums.

(c) Formulas for sums of positive integers.

(#26) Definite Integrals

(a) Net area.

(b) General Riemann sum.

(c) Definition of definite integral; Integrable functions.

(d) Reversing limits; Properties of definite integrals.

(e) Evaluating definite integrals using limits in Riemann sums.

(#27) Fundamental Theorem of Calculus

(a) Area function (Accumulator function).

(b) Fundamental Theorem of Calculus and the inverse relationship between differentiation and integration.

(c) Derivatives of area functions.

(#28) Working with Integrals

(a) Integrating even and odd functions.

(b) Average value of a function.

(c) Mean Value Theorem for Integrals.

(#29) Substitution Rule

(a) Change of variable; Changing limits of integration.

(b) Not all integrals yield to the Substitution Rule.

(c) Substitution techniques.

(d) Geometry of substitution.