Andrew Knyazev (Denver) 6.6.7 L 1

$T = T^* \geq 0$, $\|T\| \leq C$, $Tu = \mu u$

T compact for simplicity but it is irrelevant for this talk since it deals with the largest eigenvalues only.

$-\Delta u = \lambda u$, $u\|_{\partial\Omega} = 0$, $0 < \lambda < C$, $0 < \lambda_1 < \lambda_2 < \dots \to \infty$	$T = (-\Delta)^{-1}$	$\lambda = \frac{1}{\mu}$ etc.

$\dim \tilde{U} = n$, $R(\tilde{Q}) = \tilde{U}$, $\tilde{T} = (\tilde{Q}T)|_{\tilde{U}}$, $\tilde{T}\tilde{u} = \tilde{\mu}\tilde{u}$.

$\mu_1 - \tilde{\mu}_1 \leq$?

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$\dim H = 2$, $\dim \tilde{U} = 1$, $\tilde{\mu}_1 = \frac{(\tilde{u}, T\tilde{u})}{(\tilde{u}, \tilde{u})}$ for $\tilde{u} \in \tilde{U} \setminus \{0\}$

$\mu_2 < \mu_1$ — μ_2 $\tilde{\mu}_1$ μ_1, $0 \leq \mu_1 - \tilde{\mu}_1$

$$\frac{\mu_1 - \tilde{\mu}_1}{\mu_1 - \mu_2} = \sin^2 \angle(u_1, \tilde{u}_1) = \sin^2 \angle(u_1, \tilde{U})$$

approximability of eigenfunctions

(still $\dim H = 2$)

$= \inf_{\tilde{u} \in \tilde{U}} \|u_1 - \tilde{u}\|^2_{H_0^1(\Omega)}$ for membrane

Via some shift of $T \geq 0$ we could assume that the smallest of the discrete situation is zero.

Then, $\mu_2 = 0$ and so $\frac{\mu_1 - \tilde{\mu}_1}{\mu_1} = 1 - \frac{\tilde{\mu}_1}{\mu_2}$.

$\dim \tilde{U} = n$, $j = 1, \dots, n$, $\text{Range}(Q) = \tilde{U}$, Q orth. proj.

$$0 \leq \frac{\mu_j - \tilde{\mu}_j}{\mu_j} \leq \|(1 - Q) P_{1,\dots,j}\|^2 = \sin^2 \angle\{U_{1,\dots,j}, \tilde{U}\}$$

$R(\tilde{P}_{k\dots l}) = \operatorname{span}\{u_k, \dots, u_l\}$

$\sin \measuredangle\{R(P), R(Q)\} = \sigma_1((I-Q)P)$ $\dim R(P) \leq \dim R(Q)$

Singular values, first is largest

$= \|(I-Q)P\|$

$\sin \measuredangle_i\{R(P), R(Q)\} = \sigma_{\dim R(P)-i+1}((I-Q)P)$

<u>Def:</u> $\measuredangle = \measuredangle_{max}$ (Knyazev '86)

$0 \leq \frac{\mu_j - \tilde{\mu}_j}{\mu_j}$ Then $\sin^2 \measuredangle\{u_j, \tilde{U}\}$ would be an expected upper bound but is possibly wrong. However, we want to come close to that.

$\leq \sin^2 \measuredangle_m \{U_{1\ldots j}, \tilde{U} \cap \tilde{U}^{\perp}_{1,\ldots,j+m}\}$

for any $m = 1, \ldots, j$.

For $m = j$ we get the largest angle and hence this is a generalisation of that before.

No assumption on clusters of eigenvalues etc.

$\|\tilde{P}_i P_j\| \leq \frac{\|T\tilde{u}_i - \tilde{\mu}_i \tilde{u}_i\|}{|\tilde{\mu}_i - \mu_j|} \sin \measuredangle\{u_j, \tilde{U}\}$ Math Comp '97

$(1+r) \, \|(I-Q)P_{j-m+1,\ldots,j}\|$ with r

Final result

$0 \leq \frac{\mu_i - \tilde{\mu}_i}{\mu_i} \leq \ldots (1+r) \cdots$ with r from terms above