

ANALYSIS OF TRANSMISSION PROBLEMS ON LIPSCHITZ BOUNDARIES IN STRONGER NORMS

A. V. Knyazev^a

^aDepartment of Mathematics
The University of Colorado Denver
P.O. Box 173364, Campus Box 170, Denver, CO 80217-3364
<http://math.cudenver.edu/~aknyazev/>
andwer.knyazev@cudenver.edu

We consider in [1,2] a model homogeneous Dirichlet problem for a diffusion equation on a Lipschitz simply connected bounded domain with a small diffusion coefficient in a Lipschitz simply connected subdomain located strictly inside of the main domain. We study asymptotic properties of the solution with respect to the small diffusion coefficient vanishing. It is known that the solution asymptotically turns into a solution of a corresponding diffusion equation in a cavity, i.e. with Neumann boundary conditions on the inner part of the boundary. One typical proof technique of this fact utilizes a reduction of the problem to the boundary of the inner subdomain, using a transmission condition. An analogous approach appears in studying domain decomposition methods without overlap, reducing the investigation to the surface that separates the subdomains and in theoretical foundation of a fictitious domain method, e.g., to prove a classical estimate that guarantees convergence of the solution of the fictitious domain problem to the solution in the cavity [3].

On a continuous level, the analysis is usually performed in a trace norm, e.g., the $H^{1/2}$ norm for second order elliptic equations. This norm appears naturally for Poincare-Steklov operators, which are convenient to employ to formulate the transmission condition. A regularity theory of Poincare-Steklov operators for Lipschitz domains is closely related to regularity of transmission and diffraction problems and properties of layer potentials on nonsmooth interfaces, and has been extensively studied. Taking advantage of the recent progress in the regularity theory for Lipschitz domains, we provide in [1] an analysis of the transmission condition in the $H^{1/2+\alpha}$ norm with $\alpha > 0$ for a simple model problem. This result leads to a convergence theory of the fictitious domain method for a second order elliptic PDE in the $H^{1+\alpha}$ norm, while the classical result is in the H^1 norm. Here, $\alpha < 1/2$ is required for the case of Lipschitz domains we consider. For smooth domains, a similar result, but for any $\alpha > 0$, is given in [3] using a different technique.

Our proof relies on the regularity of the solution of the diffusion equation uniformly in the value of the small diffusion coefficient in the subdomain, established in [2]. The solution is regular away from the boundary and the interface of the jump in the diffusion coefficient for a sufficiently smooth right-hand side. Under our simplifying assumption that the interface of the jump is strictly inside, the question of regularity can be reduced to studying layer potentials on the interface without boundary conditions, since the interface is then is a closed Lipschitz curve without selfintersection and it does not have any common (junction) points with the boundary, where the homogeneous Dirichlet boundary condition is enforced. We show in [2] that the solution belongs to the space $H^{1+\alpha}$ with $\alpha < 1/2$ uniformly in the small diffusion coefficient vanishing in the inner subdomain.

References

- [1] A. V. Knyazev, "Analysis of transmission problems on Lipschitz boundaries in stronger norms," *Journal of Numerical Mathematics*, v. 11, no. 3, 225-234, 2003.
- [2] A. V. Knyazev and O. Widlund, "Lavrentiev regularization + Ritz approximation = uniform finite element error estimates for differential equations with rough coefficients," *Mathematics of Computation*, v. 72, 17-40, 2003.
- [3] V. D. Kopchenov, "A method of fictitious domains for the second and third boundary value problems," *Trudy Mat. Inst. Steklov*, v. 131, no. 246, 119-127, 1974.