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Department of Mathematics
Numerical Linear Algebra (MATH or CSC6664)
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Report on Spectral Properties of the Matrices
Connected With the Graphs of Images
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1 INTRODUCTION

The idea of the project is to compute the smallest and largest eigenvalues using a large-scale picture by saving the Spectral segmentation and N-Cuts matrices for different scaling factors. An in depth analysis has been made to see the growth of the condition numbers as a function of the matrix size. This paper holds the brief report of all these results including a report on timing and memory use for each run and gives the actual segmented images for individual image resolution experiments.

2 ANALYSIS

The photo we have used is from the European Southern Observatory (ESO) website "www.eso.org" and comes with permission for reproduction provided the ESO is credited. The photograph is of the "Very Large Telescope (VLT) at Paranal" "in the Atacama desert region in Chile."

The code we have used is located at www-math.cudenver.edu/~cjameson/6664/SegAnalysisDemo. The original tar-files from Leo Grady are there along with the set of extracted m-files necessary to run his program segmentationCompareDemo.m. We have renamed four files, three of them by appending the letter D to the front of the original name. Several lines of code were added to partitiongraph.m to gather information to analyze the

performance of the eigen-solver. `partitiongraph.m`, `recursivepartition.m`, and `imgsegment.m` are the three subroutine filenames to which the letter D has been appended. The latter two were modified only by adding the letter D to the appropriate function calls. They are part of the calling chain for `partitiongraph.m`.

The program `segmentationCompareDemo.m` has been so heavily modified that it no longer resembles the original code, but almost all of its original code has been used either in the program we wrote to replace it, `segDemoAnalysis.m`, or in three other routines we wrote to display output.

`segDemoAnalysis.m` essentially, takes Leo Grady's original code, deletes the isoperimetric segmentation method, and applies the Spectral and Normalized Cuts methods to a series of different resizings of the same image. It saves all output to disk as the segmentations for each size are completed. If the segmentations of all sizes completes, the program then calls two of our three display subroutines, `dispGraphs.m`, `dispAllOutput.m`, and `dispOutputxType.m`. `dispGraphs` displays performance information on the eigen-solver. The other two display the segmentation outputs. If the segmentations of all sizes does not complete (i.e. the program crashes), `segDemoAnalysis.m` can be run without calling the segmentation subroutines. It then displays the output from the previous run. For our report, we used a picture with 8,238,000 pixels. The size range of our partition run was from 8,238,000 to 6 pixels. The number of pixels and the size of the matrix fed into `eigs` are always the same. All runs were made in Godzilla. Obtaining memory usage data was a non-scientific process. The runs were made late in the evening and monitored using the program "Tops". We modified the original code so that each iteration would calculate and record for the one matrix size of the largest and smallest eigen values, and their respective calculation times, for the Spectral and N-Cuts methods. The iteration for the largest two pixel sizes showed a pattern of memory usage. Finding the smallest eigen value for each method took roughly four times as much memory as finding the respective largest eigen value. Also, the N-Cuts method took at least a third more memory than the Spectral method for each eigen value.

Here are tables representing the pixel size and memory usage of the runs for the smallest and largest eigen values:

Table1 On Godzilla Memory Usage for all sizes

Marrix Size	Memory Usage
less than 1000	227 Mbytes
4, 818	236 Mbytes
25,232	302 Mbytes
48,530	391 Mbytes
101,565	615 Mbytes
228,500	1100 Mbytes

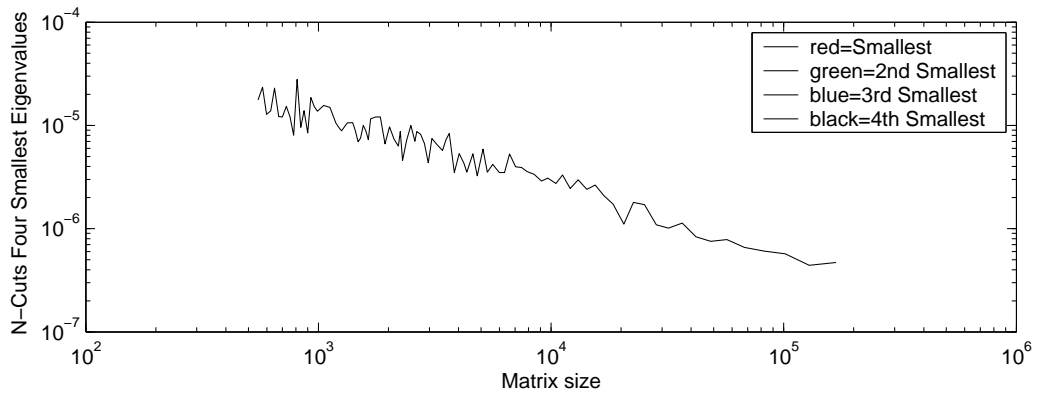
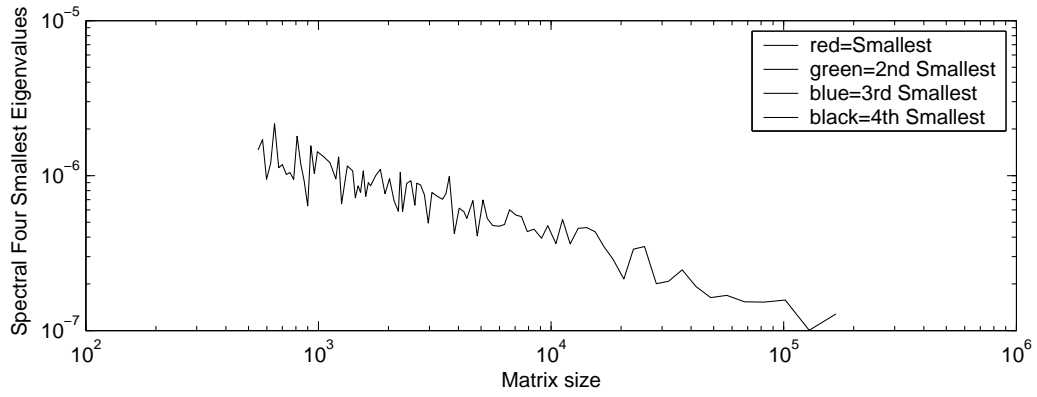
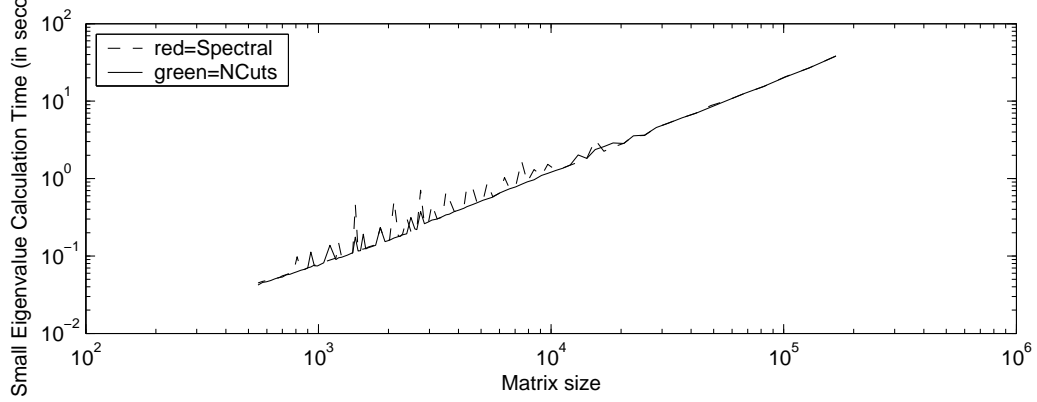
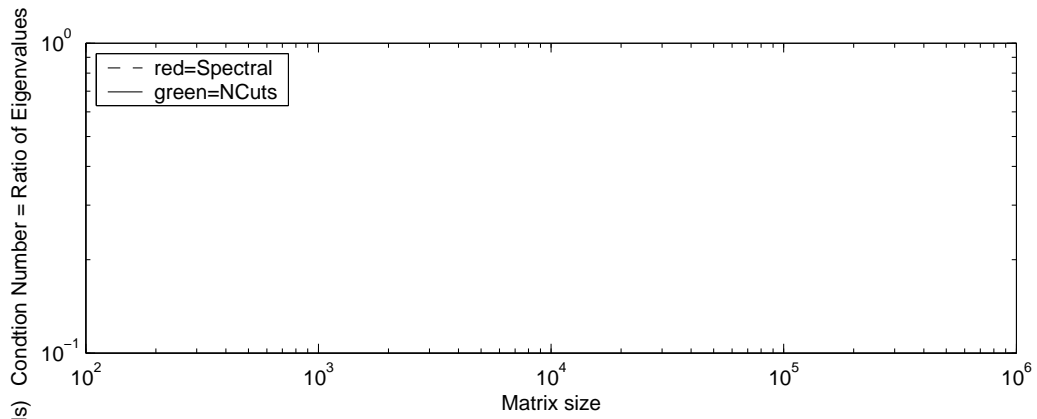
Please note here: the memory usage for the Spectral and NCuts methods is the same.

In general the formula for the condition number is Largest Eigen Value/Smallest Eigen value. Thus, the condition number for the Spectral and N-CUT methods is computed using the LARGEST EIGEN VALUE divide by the SMALLEST EIGEN VALUE.

INITIALIZATION

For all runs, nothing in this section was changed.

Tolerance: For our baseline runs, we used machine precision (Toler = eps) for the tolerance. At this tolerance, we made one run with a 4-node connectivity (Connect = 0) and one run with 8-node connectivity (Connect = 1). In both of these runs, we calculated large eigenvalues (Calc LE = Yes). (Note that in all runs that actually calculate, "runSegment" must be set to "Yes", otherwise segDemoAnalysis uses a file to generate output). We made two runs to try to find the most consistent partition across the spectrum of picture resizings. For Spectral Cuts, our best run was with Connect = 1 and Toler = 1e-15. For Normalized Cuts, it was Connect = 0 and Toler = 1e-12. In both of these runs, we chose not to calculate large eigenvalues (Calc LE = No). In the first two runs, we also opted to calculate the four smallest eigenvalues. In the second two runs, we calculate only the smallest. Thus, in our output, the first two runs include the condition numbers and the four smallest eigenvalues for each matrix size, the second two runs do not. Please see the details of the following figures for in depth comparison and analysis of the runs output of the Spectral and NCuts methods.



Spectral Outline:3835pixels Spectral Outline:4216pixels Spectral Outline:4615pixels Spectral Outline:5100pixels Spectral Outline:5616pixels



Spectral Outline:6308pixels Spectral Outline:7040pixels Spectral Outline:7905pixels Spectral Outline:9100pixels Spectral Outline:10486pixels



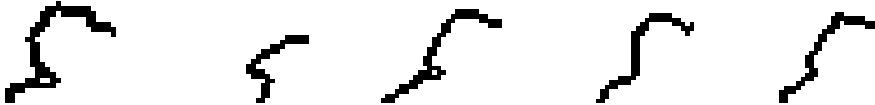
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Spectral Outline:31975pixels Spectral Outline:41940pixels Spectral Outline:57000pixels Spectral Outline:82200pixels Spectral Outline:128625pixels



Spectral Outline:5750pixels Spectral Outline:6240pixels Spectral Outline:6750pixels Spectral Outline:7280pixels Spectral Outline:7830pixels



Spectral Outline:8400pixels Spectral Outline:8990pixels Spectral Outline:9600pixels Spectral Outline:10540pixels Spectral Outline:11880pixels



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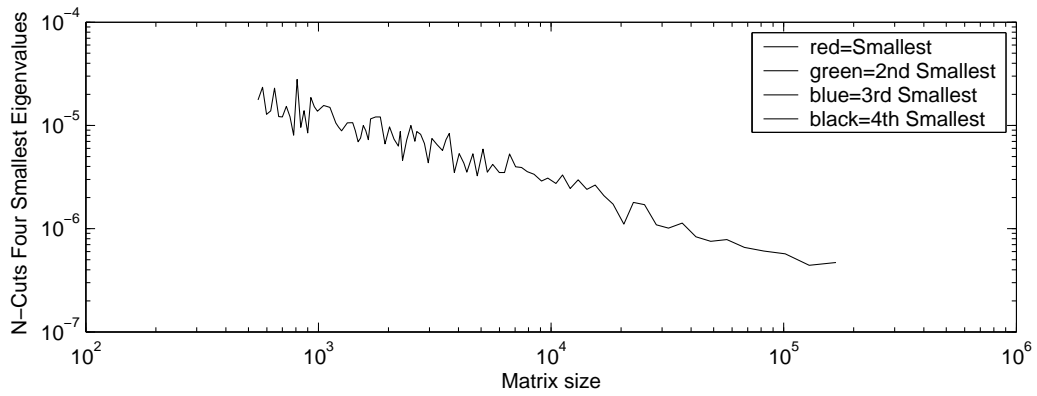
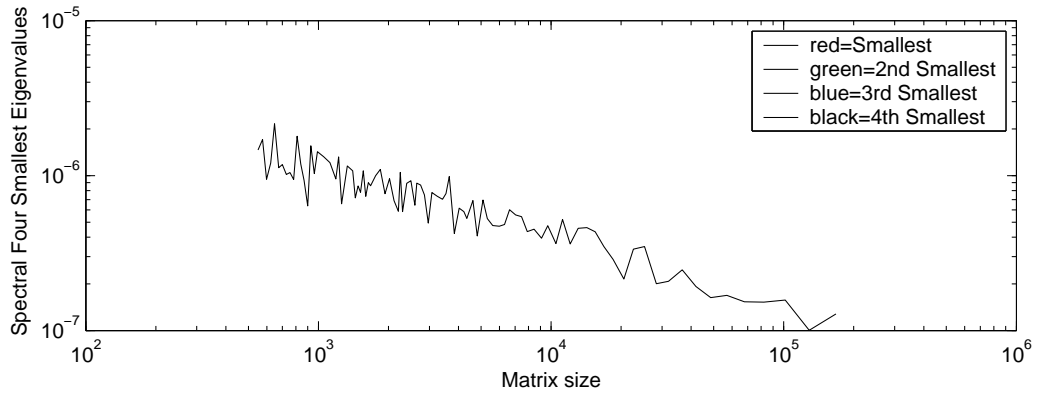
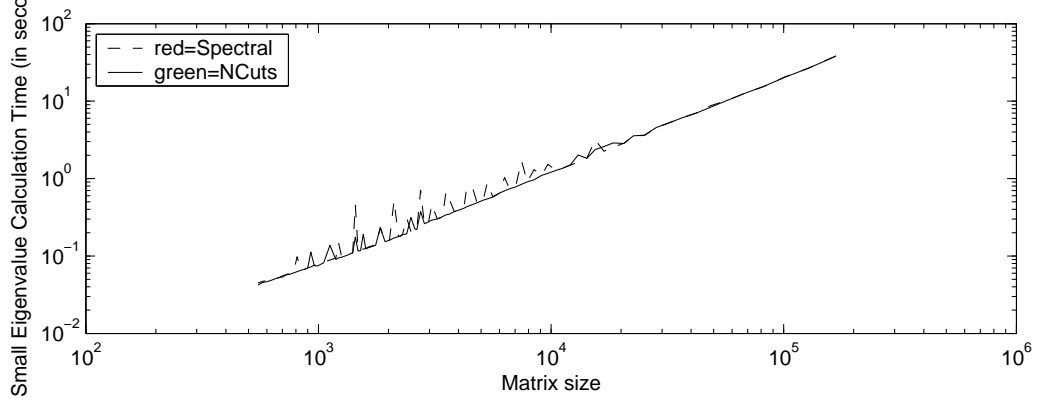
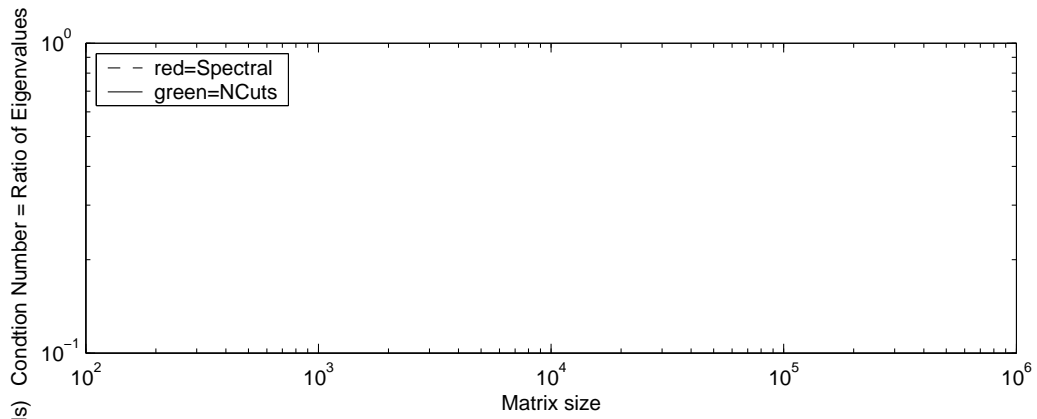


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Spectral Outline:25970pixels Spectral Outline:27500pixels Spectral Outline:29640pixels Spectral Outline:32400pixels Spectral Outline:35340pixels





Ncuts Outline:3835pixels Ncuts Outline:4216pixels Ncuts Outline:4615pixels Ncuts Outline:5100pixels Ncuts Outline:5616pixels



Ncuts Outline:6308pixels Ncuts Outline:7040pixels Ncuts Outline:7905pixels Ncuts Outline:9100pixels Ncuts Outline:10486pixels



Ncuts Outline:12075pixels Ncuts Outline:14250pixels Ncuts Outline:16864pixels Ncuts Outline:20550pixels Ncuts Outline:25232pixels



Ncuts Outline:31977pixels Ncuts Outline:41944pixels Ncuts Outline:57000pixels Ncuts Outline:82200pixels Ncuts Outline:128625pixels



Ncuts Outline:575pixels Ncuts Outline:624pixels Ncuts Outline:675pixels Ncuts Outline:728pixels Ncuts Outline:783pixels



Ncuts Outline:840pixels Ncuts Outline:899pixels Ncuts Outline:960pixels Ncuts Outline:1054pixels Ncuts Outline:1188pixels



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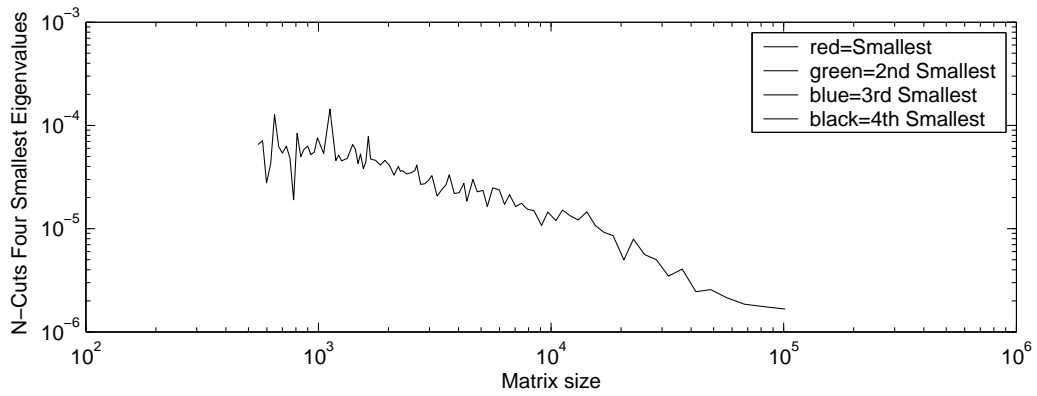
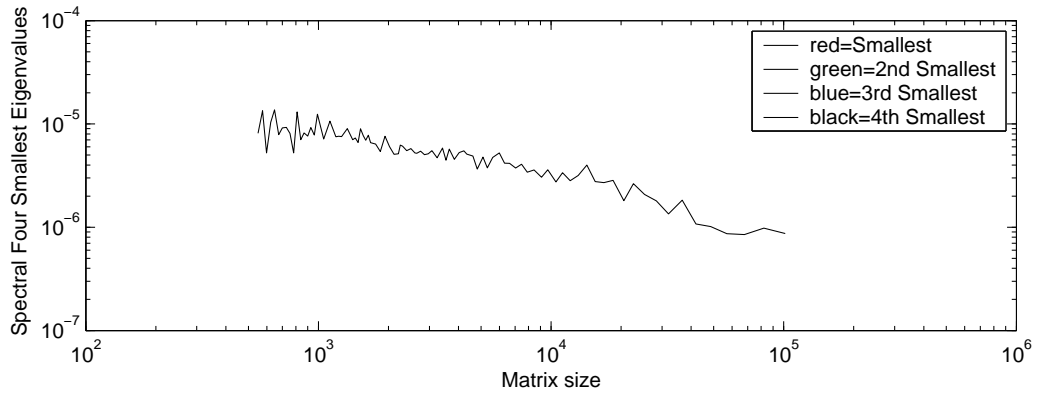
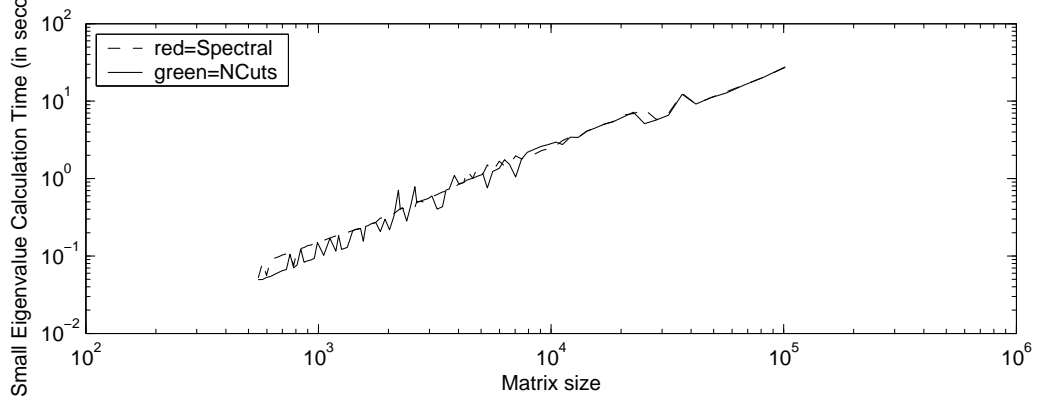
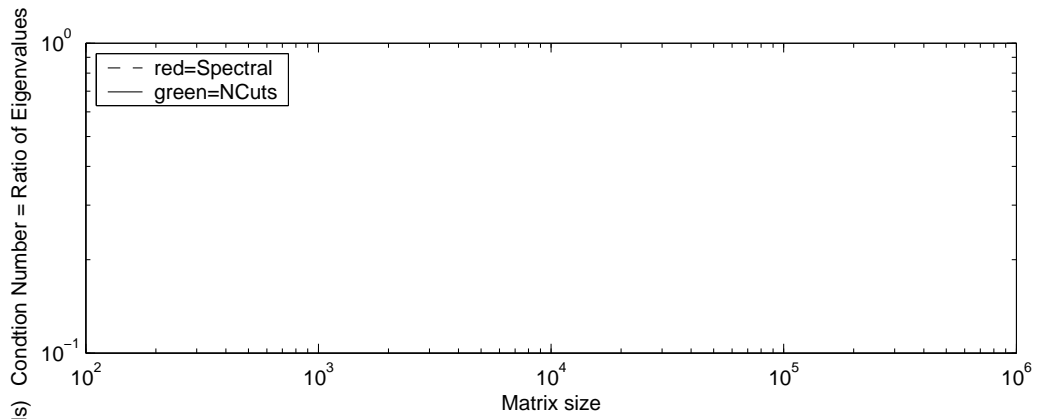


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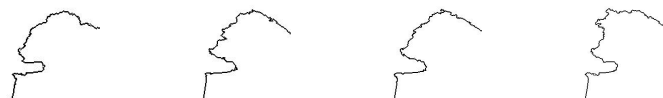
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Spectral Outline:3197 Spectral Outline:4194 Spectral Outline:5700 Spectral Outline:8220pixels



Spectral Outline:575 Spectral Outline:624 Spectral Outline:675 Spectral Outline:728 Spectral Outline:783pixels



Spectral Outline:840 Spectral Outline:899 Spectral Outline:960 Spectral Outline:1054 Spectral Outline:1188pixels



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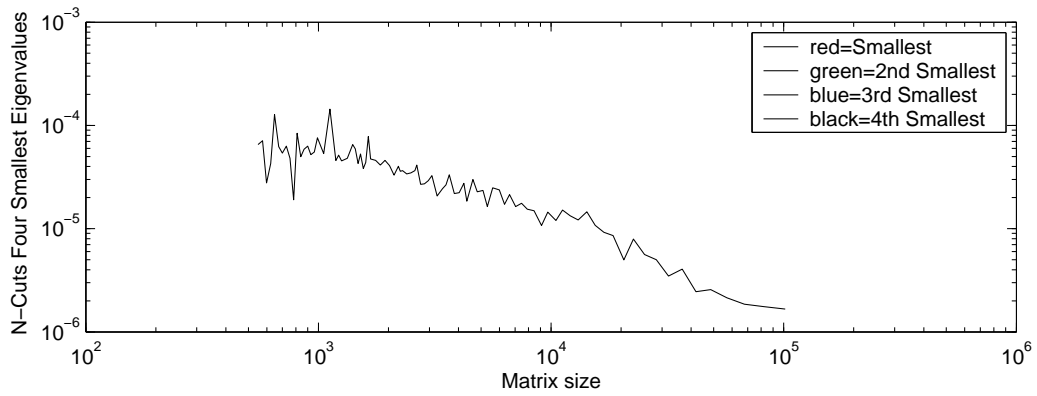
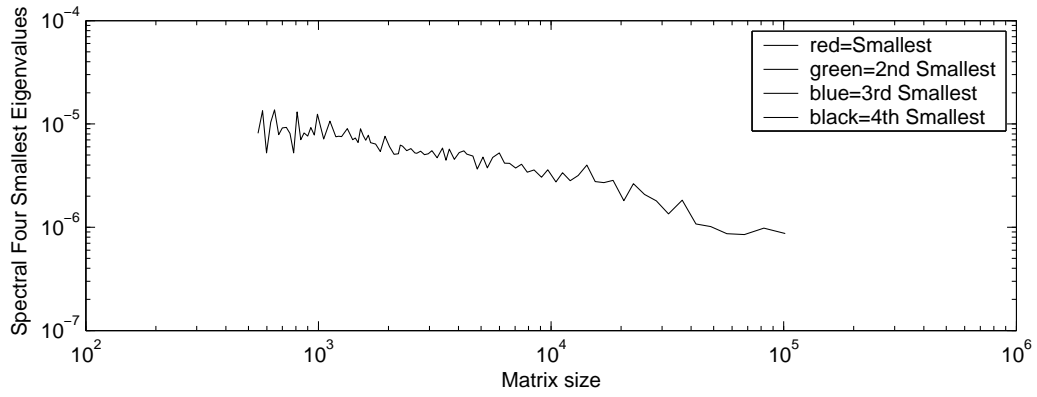
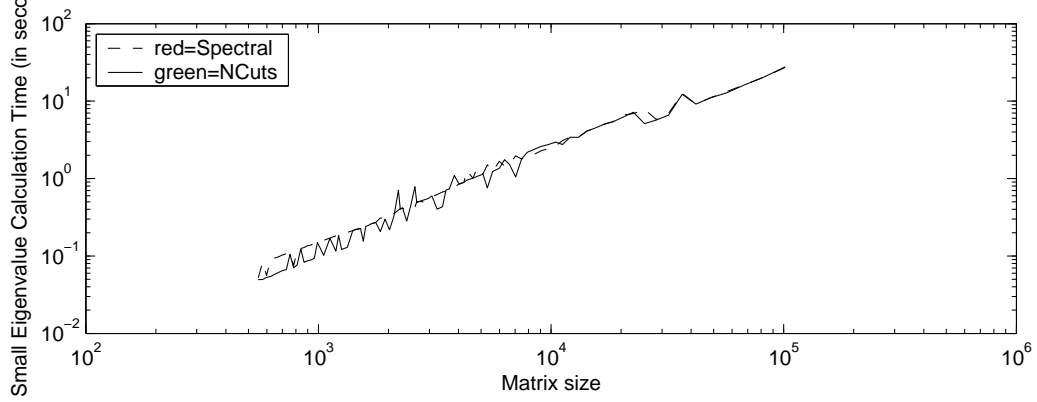
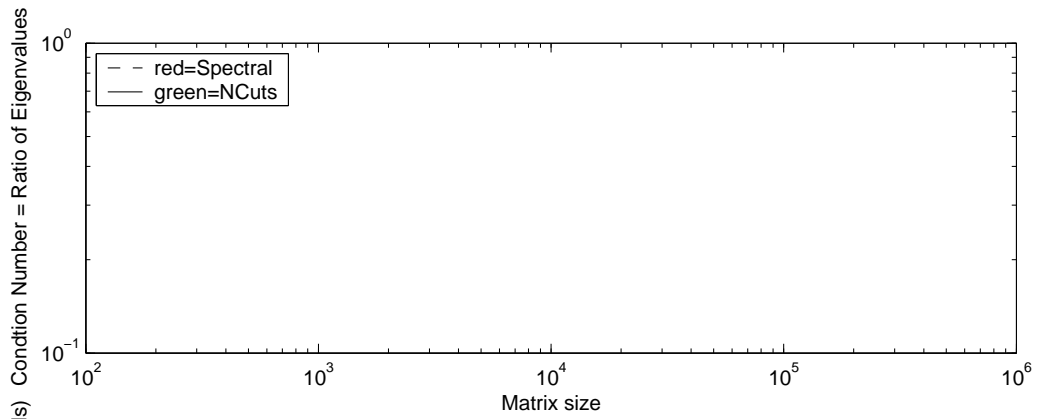


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Spectral Outline:2597 Spectral Outline:2750 Spectral Outline:2964 Spectral Outline:3240 Spectral Outline:3534pixels





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