On Some Spectral Properties of the Matrices Connected With the Graphs of Images

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Abstract. The spectral properties of the laplacian and normalized laplacian matrices are considered. These matrices are related to the graphs of the images. The bahavior of the condition numbers of these matrices in the subspaces orthogonal to the first eigenvectors is studied. The question of time needed for calculation of some eigenvalues is investigated.

Key words. Graph, Fiedler's vector, eigenvalue, eigenvector, condition number, spectral partitioning, NCuts partitioning

AMS(MOS) subject classifications. 15A12, 15A18, 65F15, 65F35, 65F50, 68R10, 05C40, 05C50, 05C90

1 What Images We Have Considered

The results presented in this report are related to the picture of European telescope. The size of the initial image is $2746 \times 3000 \times 3$, that is presented in a `MATLAB` code as 197712000 double array.

2 What Methods We Used

In order to study the behavior of several spectral characteristics of the matrices related to the graphs of images the partitioning was executed for different sizes of images by 2 methods - spectral partitioning and NCuts partitioning. The first one is based on the results of M. Fiedler. Following that results the second eigenvector (called Fiedler's vector) is good characteristic of the connectness of the graph. For both of given methods are considered also two different cases - one of them defines the graph geometry by the 4-point pattern, and second one - by the 8-point pattern. The images of different sizes were obtained by resizing of the original image using the `MATLAB` command `imresize`, namely by dividing of the X and Y sizes of original image by the sequence of some numbers. These sequences may be obtained in different ways, that depend on our purposes. In this report the case is considered, when this sequence is power sequence (the geometric progression). The first term `nBegin`, last term `nLast` and the number of images `nPictures`, we need to partition, are defined by ourself. By these initial data common quotient of the progression `resizemult` is defined.

3 What We Have Got

The computational experience showed, that for the `nBegin` $= 4$ (the size of laplacian matrix is 514500×514500) `MATLAB` code working on GODZILLA computer makes the spectral and NCuts segmentation for 4-point connection using 1.9GB and 2.05GB of memory respectively. For 8-point connection spectral partitioning also works (using the memory 2.43GB) but the NCuts goes out of memory. Namely, code crushed because of insufficiency of memory, when calculating the smallest

NN	Size of Matrix	Spectral -4	NCUT - 4	Spectral - 8	NCUT - 8
1	25,232	100 MB	180 MB	145 MB	150 MB
2	48,530	170 MB	255 MB	217 MB	240 MB
3	101,565	360 MB	420 MB	453 MB	480 MB
4	128,625	450 MB	520 MB	560 MB	605 MB
5	329,400	1.17 GB	1.28 GB	1.5 GB	1.6 GB
6	514,500	1.9 GB	2.05 GB	2.43 GB	Out of Memory

Table 1: Computer Memory Usage

eigenvalues of the matrix $L2$ by `MATLAB` function `eigs`. The results of using the computer memory for some sizes of matrices are given in the Table 1.

For `nBegin` $= 5$, `nLast` $= 512$, `nPictures` $= 50$, the code works about 75 minutes and calculates all the needed quantities.

The pictures given in this report are related to the case when `nBegin` $= 8$, `nLast` $= 128$, `nPictures` $= 20$. This choice gives the matrices of sizes from 483×483 to 128625×128625. This problem tooks about 15 minutes of the CPU time on GODZILLA. Other results, that is the dependences of the condition numbers in subspaces, of the time for smallest eigenvalues, of few small eigenvalues on the sizes of matrices, are obtained for the following initial data: `nBegin` $= 5$, `nLast` $= 128$, `nPictures` $= 50$.

We studied the behavior of the condition numbers of the laplacian and normalized laplacian (NCuts) matrices L and $L2$ respectively for the initial values `scaleIso` $=$ `scaleNcuts` $= 100$. Condition number in our case means the ratio of the largest and smallest positive eigenvalues (the condition number in the subspaces, orthogonal to first eigenvector), because the smallest eigenvalues of both matrices are zero. The eigenvalues were calculated using the `MATLAB` function `eigs`. The related results in the graphical view are given below.

We studied also the CPU time needed for calculation of the smallest and largest eigenvalues of matrices considered in the project. The graphics showing the dependence of the mentioned time on the sizes of the matrices are given below. These times were calculated by two different ways - using `tic`, `toc` and `cputime` functions. The results given below are obtained by the function `cputime`.

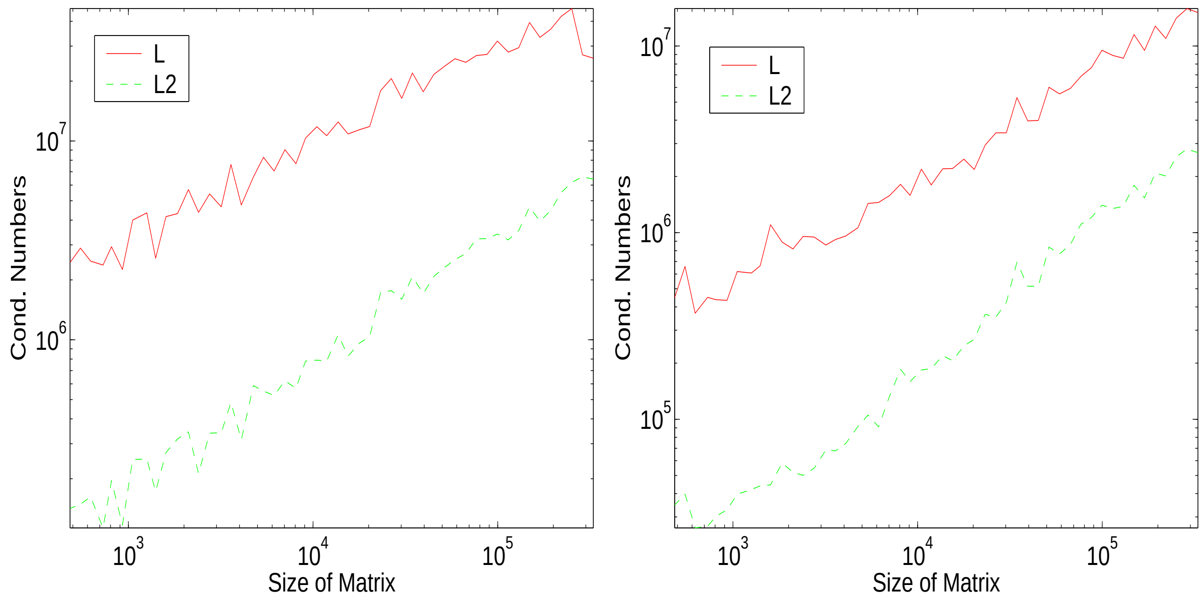

Figure 1: Condition Numbers, 4-Point - left, 8-Point - right

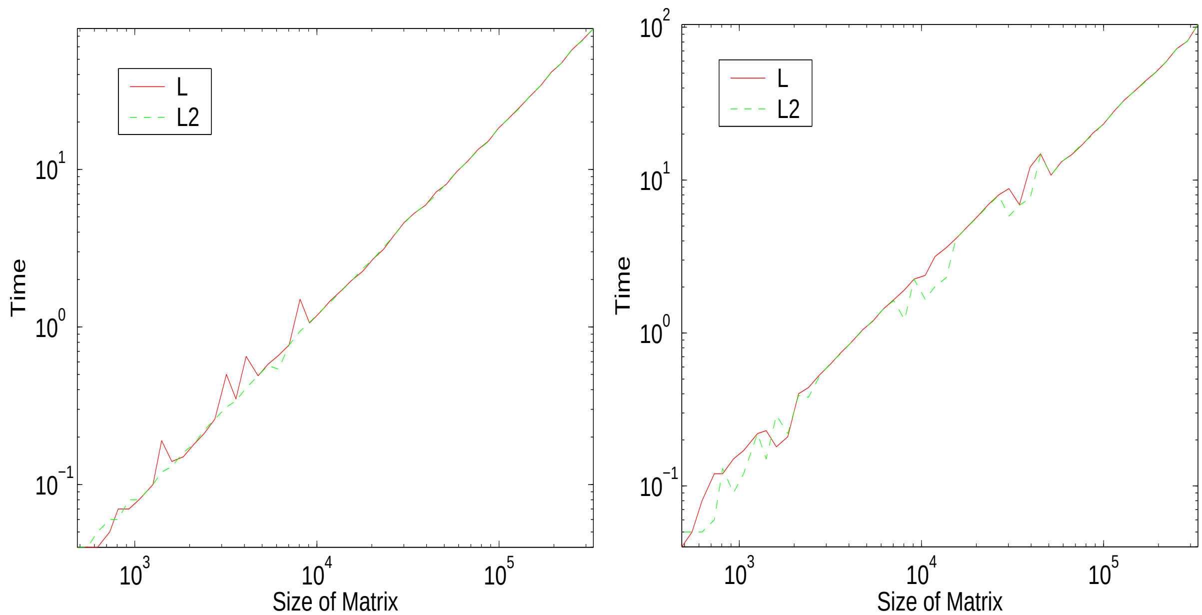

Figure 2: Time for Smallest Eigenvalues, 4-Point - left, 8-Point - right

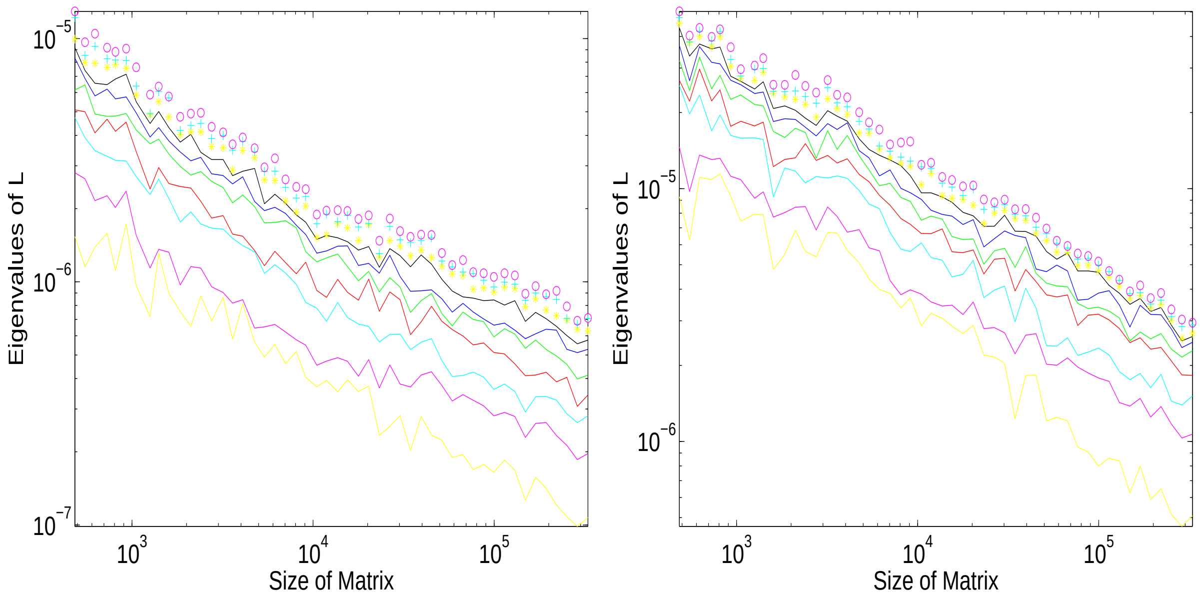

Figure 3: 10 Small Eigenvalues of L, 4-Point - left, 8-Point - right

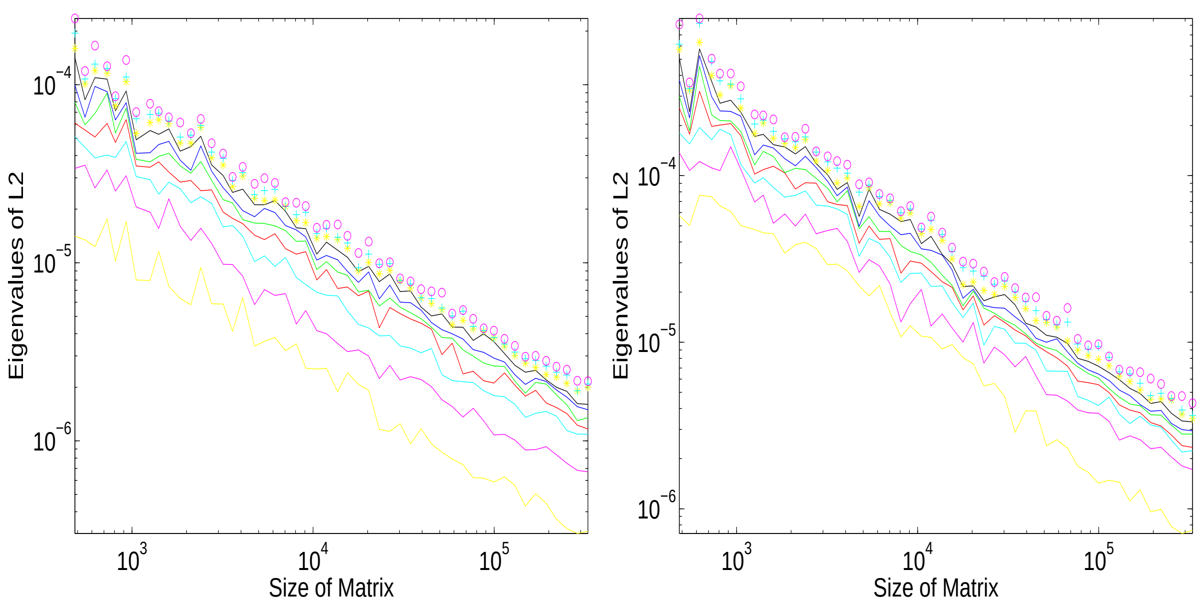

Figure 4: 10 Small Eigenvalues of L2, 4-Point - left, 8-Point - right

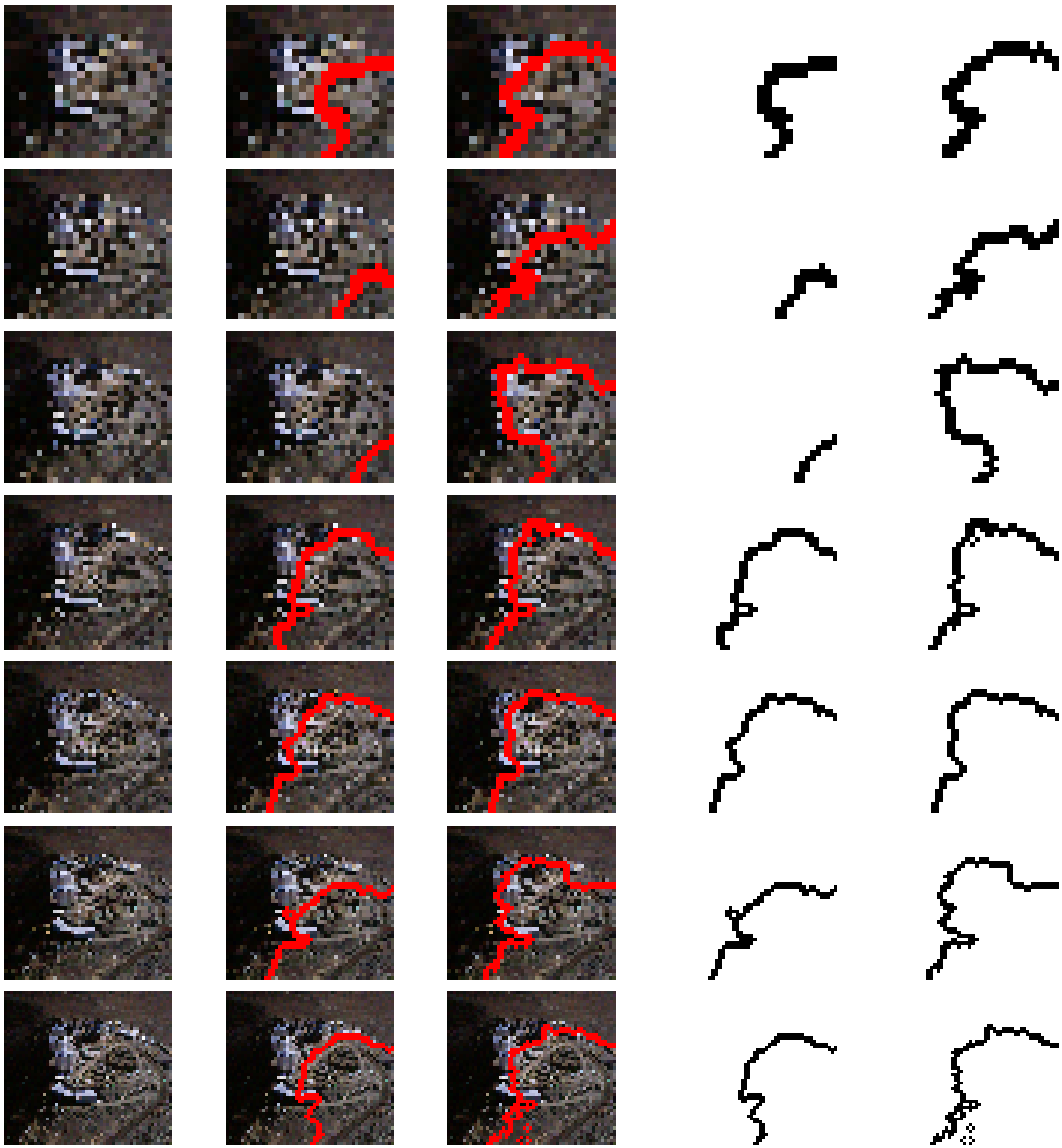

Figure 5: Partitioning for 4-Point Pattern, Pictures 1-7.
Original Image - Spectral Partition. - NCUT Partition.- Spectral Line - NCUT Line
Sizes of images (pixels): 483, 648, 868, 1188, 1599, 2112, 2856

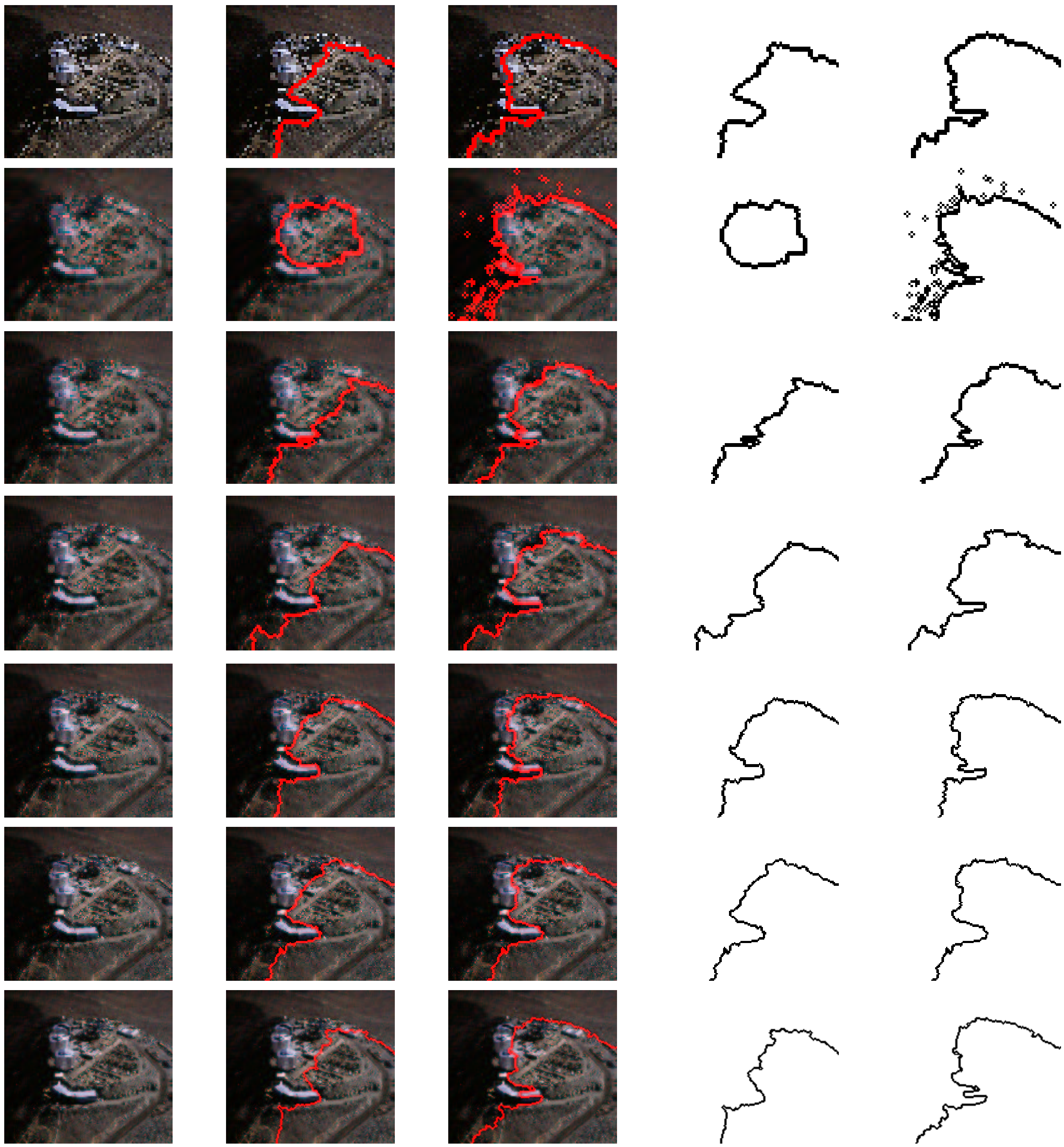

Figure 6: Partitioning for 4-Point Pattern, Pictures 8-14.
Original Image - Spectral Partition. - NCUT Partition.- Spectral Line - NCUT Line.
Sizes of images (pixels): 3835, 5100, 6873, 9200, 12296, 16605, 22152

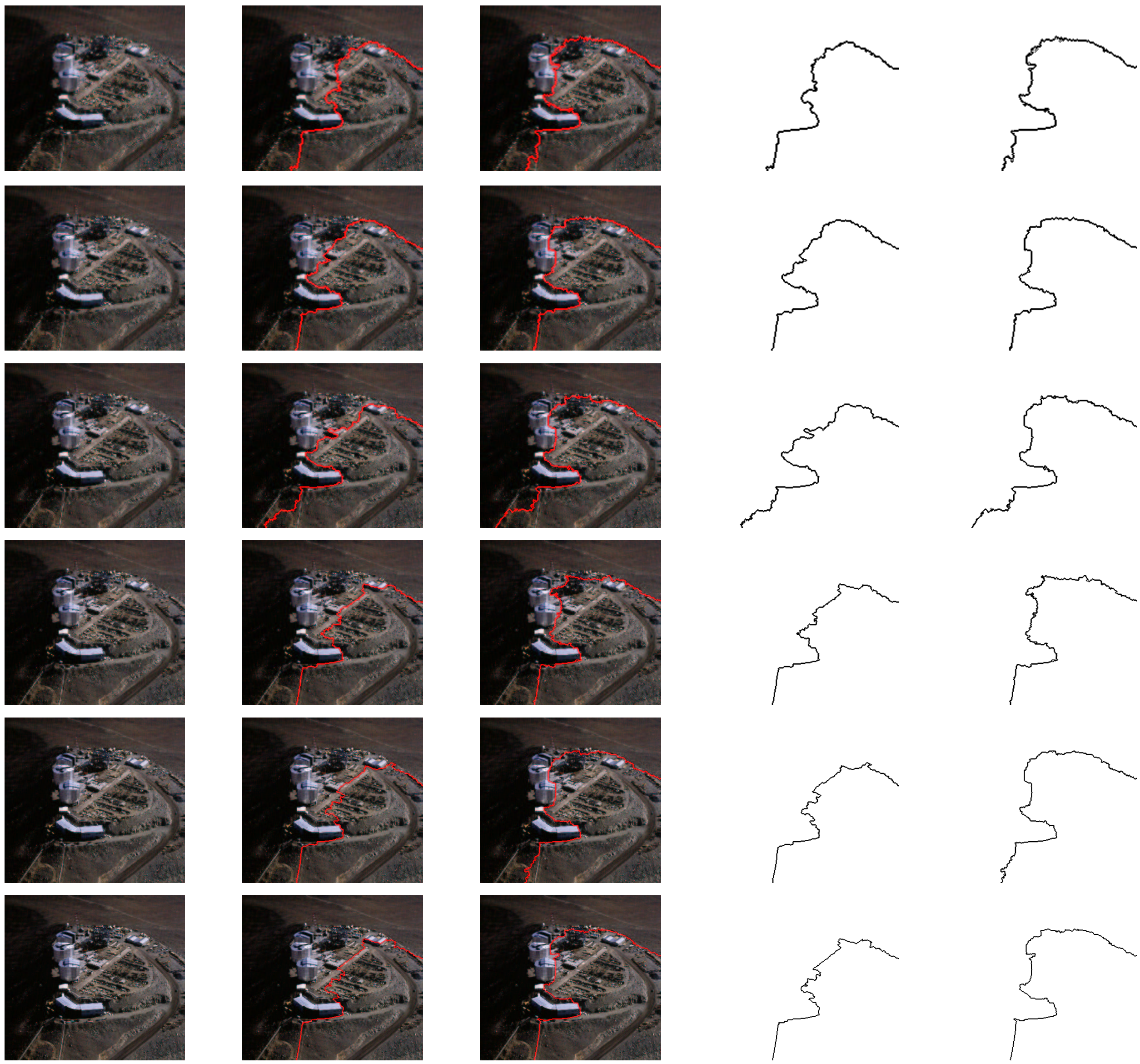

Figure 7: Partitioning for 4-Point Pattern, Pictures 15-20.
Original Image - Spectral Partition. - NCUT Partition.- Spectral Line - NCUT Line.
Sizes of images (pixels): 29700, 39919, 53482, 71680, 95904, 128625

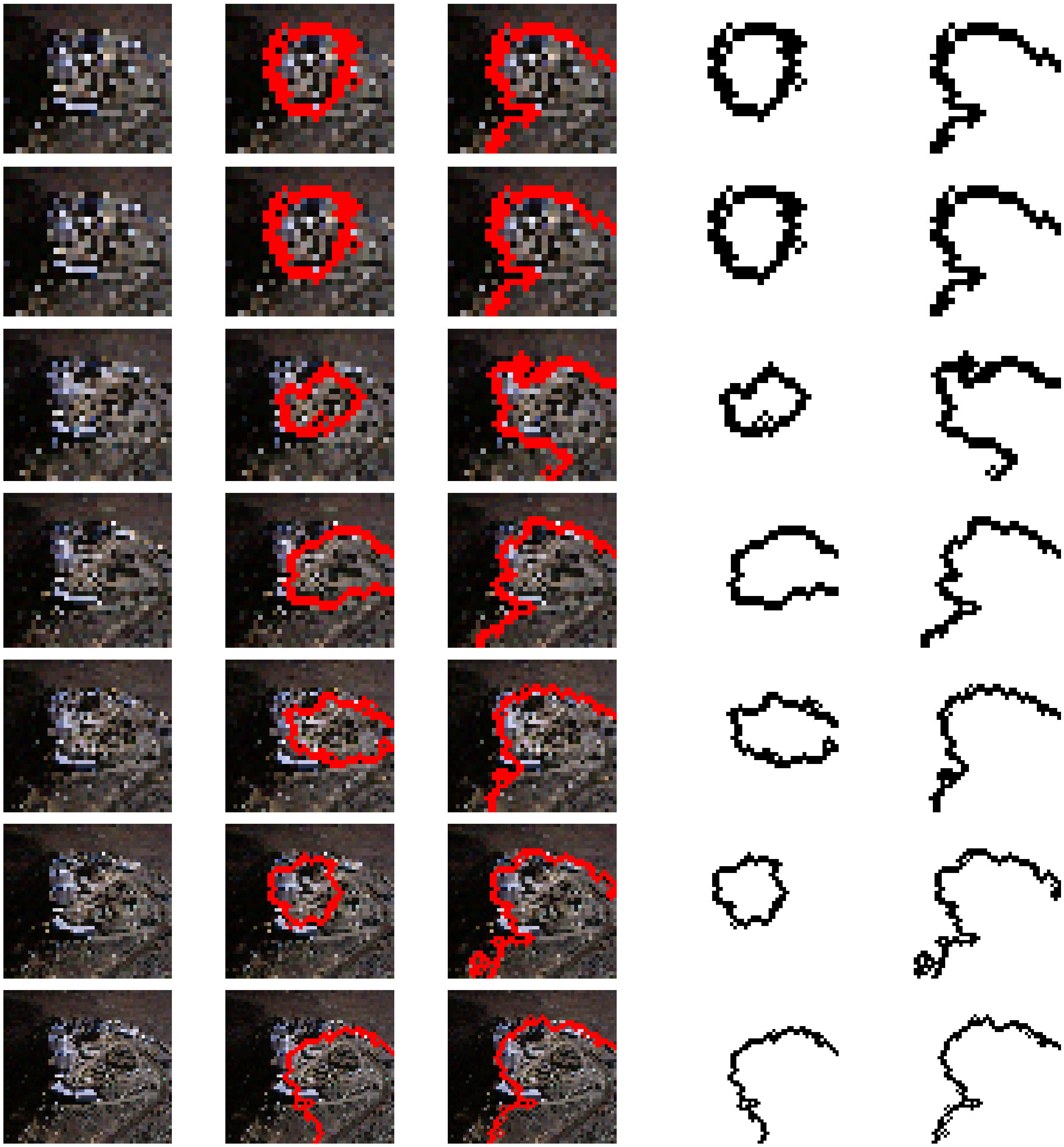

Figure 8: Partitioning for 8-Point Pattern, Pictures 1-7.
Original Image - Spectral Partition. - NCUT Partition.- Spectral Line - NCUT Line.
Sizes of images (pixels): 483, 648, 868, 1188, 1599, 2112, 2856

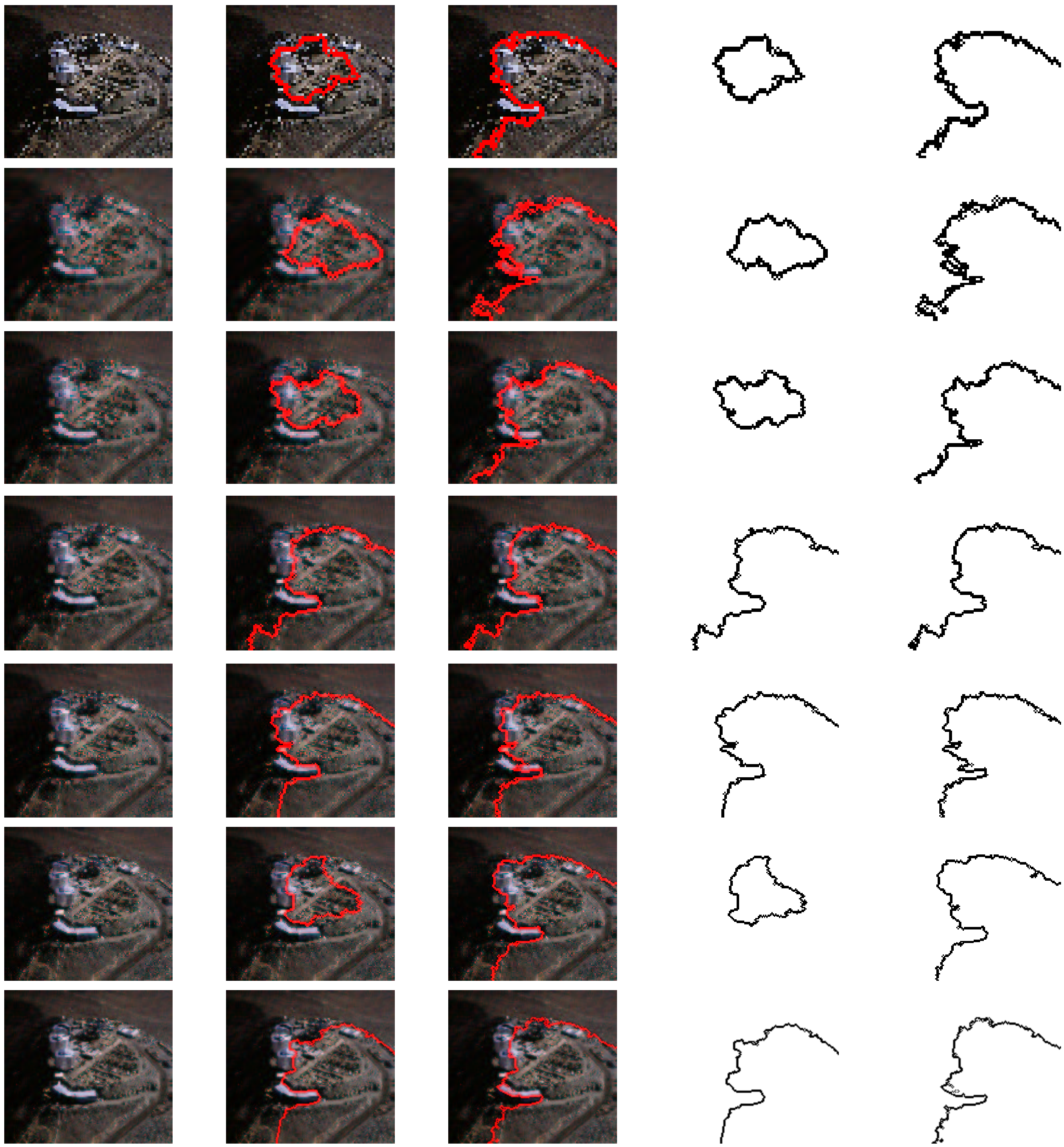

Figure 9: Partitioning for 8-Point Pattern, Pictures 8-14.
Original Image - Spectral Partition. - NCUT Partition.- Spectral Line - NCUT Line.
Sizes of images (pixels): 3835, 5100, 6873, 9200, 12296, 16605, 22152

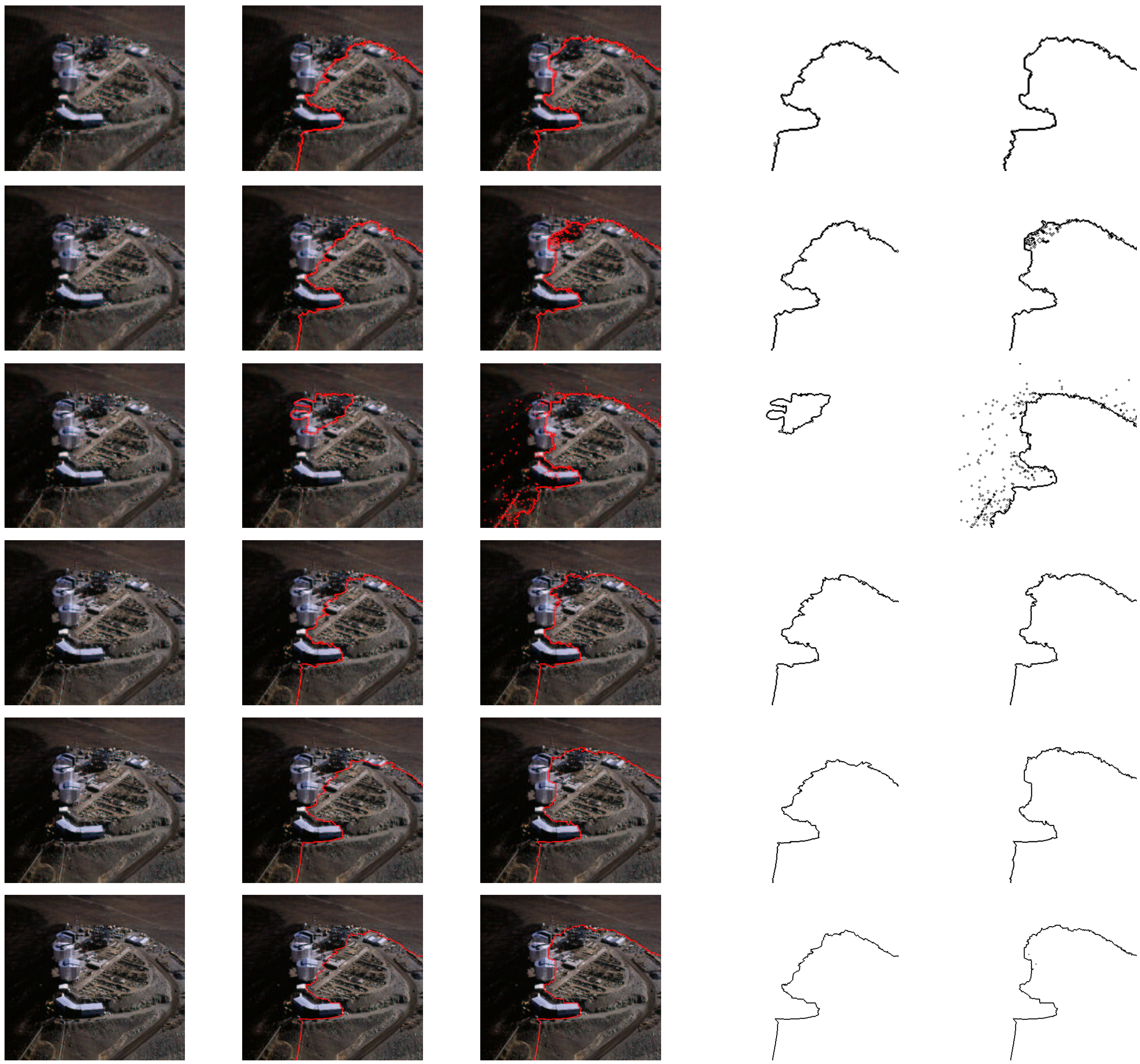

Figure 10: Partitioning for 8-Point Pattern, Pictures 15-20.
Original Image - Spectral Partition. - NCUT Partition.- Spectral Line - NCUT Line.
Sizes of images (pixels): 29700, 39919, 53482, 71680, 95904, 128625