

MATH 3200, Fall 2009 - Homework 2
(Due Thurs. Oct. 29 at the beginning of class)

Please write up solutions to the following problems. Your solutions should be neatly, and carefully written using complete sentences and correct grammar. You are encouraged to consult other students in the class, but you must write your solutions using your own words.

Note: You are free to use calculators or computers to help solve these problems. However, your writeup should clearly explain how you formulated and solved the problems.

1. Solve the following initial value problem:

$$y'' + 4y = \frac{1}{2} \cos 2t + 3e^{2t} + 2t^2, \quad y(0) = 1, \quad y'(0) = 0.$$

2. An object of mass $m = 4$ slugs is attached to a spring with spring constant $k = 20$ lb/ft. Suppose the object is released from 7 in. above its equilibrium with a downward initial velocity of 2.5 ft/s. Assuming there is no resistive force (i.e., no damping), find (a) the maximum displacement from the equilibrium position; (b) the time at which the object first passes through its equilibrium position; and (c) the period of motion. (Assume that there is no damping force).
3. An object weighing 8 lb stretches a spring 6 in. beyond its natural length. Suppose that there is a resistive force that is proportional to the velocity, with proportionality constant $c = 2$ slugs/second. Suppose the object is set into motion from its equilibrium position with an initial velocity of 1 ft/s in the downward direction.
 - (a) Determine the maximum displacement from the equilibrium position.
 - (b) What is the quasiperiod of the motion?
 - (c) For what values of c is the motion underdamped?
 - (d) Assuming $c = 2$ slugs/second, for what weights of the object is the motion underdamped?
 - (e) Suppose that $c = 6$ slugs/second and that the object is released a distance x_0 feet below the equilibrium position. What is the largest value of x_0 for which the object will not pass through the equilibrium position?
4. An object of mass 1 slug is attached to a spring with spring constant $k = 4000$ lb/ft. It is subjected to a resistive force proportional to the velocity, with proportionality constant $c = 40$ slugs/second, and is subjected to an external force $f(t) = 600 \sin t$ lbs.
 - (a) Determine the displacement of the object if the object starts at rest from the equilibrium position.
 - (b) What is the transient solution?
 - (c) What is the steady-state solution?
 - (d) Determine the envelope of the steady-state solution.