MATH 3200, Fall 2009
Review for Exam 2 (Exam will be on Thurs., Nov. 19, 2009)

General The exam will cover the material on study guides 6 through 11. (So any problem on the study guides, or similar problems may appear on the exam). The test will be closed book and closed notes. You may use a table of integrals and a calculator.

Essential Material The test will emphasize the following material.

- Terminology/Definitions: Second order linear ODE, homogeneous vs. nonhomogenous, linear combination, superposition principle (for homogeneous equations), Fundamental set of solutions, Linear independence, Wronskian, Characteristic Equation, Euler's formula, Linear System of first order ODEs, eigenvalue, eigenvector, matrix functions, vector functions, operations on matrix functions, phase plane.
- Essential Skills
  1. Solve 2nd order linear differential equations with constant coefficients.
     - Find general solution to homogenous equation.
     - Use the method of undetermined coefficients to find a particular solution to nonhomogeneous equation. (Be sure you know what to do if the right hand side is a solution to the homogeneous equation).
     - Solve an initial value problem (using initial conditions $y(t_0) = y_0$, and $y'(t_0) = v_0$.
  2. Solve a homogeneous higher order equation with constant coefficients.
  3. Formulate and solve a Spring-Mass System problem. (Know how to incorporate the spring constant, the damping coefficient, and the forcing function into the differential equation).
     - Hooke's Law
     - damping coefficient
     - Behavior: (3 cases: overdamped, critically damped, underdamped).
     - Vibrations
     - period, frequency, amplitude, phase
  4. Calculate the eigenvalues and eigenvectors of a 2×2 or 3×3 matrix.
  5. Find the general solution to a 2-dimensional homogeneous first order linear system with constant coefficients. Know what to do for each of the following cases:
     - 2 distinct real eigenvalues.
     - Repeated eigenvalue, with 2 linearly independent eigenvectors.
     - Repeated eigenvalue, with only one eigenvector.
     - Complex conjugate eigenvalues.
  6. Solve a nonhomogeneous 2-dimensional linear system with constant coefficients, of the form $\mathbf{X}' = A\mathbf{X} + \mathbf{F}(t)$, using the method of undetermined coefficients to find a particular solution.
     - Assume the form of the solution: $\mathbf{X}(t) = \mathbf{a}f_1(t) + \mathbf{b}f_2(t) + ...$, where $f_1(t), f_2(t)$, etc. are functions related to components of the function $\mathbf{F}(t)$. (NOTE: this is similar to the method used in Chapter 4, except that the coefficients of the function $f_1, f_2, ...$ are **vectors** instead of scalars.
     - Know what to do if one the components of $\mathbf{F}$ is also a component in the solution to the corresponding homogeneous system. (NOTE: You can't just multiply your guesses by t–this is slightly different than what we did in chapter 4.). Example: If one component of $\mathbf{F}(t)$ is e^{2t}, and if $\mathbf{c}e^{2t}$ is a solution to the homogeneous equation, then the particular solution would need to be of the form: $\mathbf{a}te^{2t} + \mathbf{b}e^{2t} + ...$ That is, it includes terms involving both te^{2t} *and* e^{2t}.)

7. Use the eigenvalues of a system to determine the qualitative behavior of solutions, and classify the equilibrium points.

Essential Problems

- Section 4.4: 11,13,15, 31
- Section 4.5: 27, 33, 35, 37, 47
- Section 5.1: 5, 7, 11, 13
- Section 5.2: 5, 7, 9, 11, 17
- Section 6.2: 25, 27, 29, 33
- Section 6.3: 15, 17, 21, 23
- Section 6.4: 1, 5, 7, 15, 23, 25, 33, 35
- Section 6.5: 1,3,7
- Section 6.6: 1,3,4,5

Other Stuff

- Calculate the Wronskian of a set of n solutions to a linear system of first order ODEs. (for $n = 2$ or $n = 3$).
- Operations on matrix functions.
  - Equality, addition, scalar multiplication, matrix multiplication, inversion.
  - Limits and Derivatives.
  - Antiderivatives of matrix functions.
  - Representing linear systems in matrix terms.
  - Existence and Uniquesness theorem for first order linear systems.
  - Rewriting an nth order scalar linear equation as a first order system.
- Extensions of basic terminology to linear systems: homogeneous, superposition principle, fundamental sets, general solution.
- Fundamental matrix.