MATH 3200, Fall 2009
Review for Final Exam (Exam will be on Tues. Dec. 8, 1-3 pm)

General The exam will cover all of the material covered in study guides 1–11. The test will be closed book and closed notes. You may use a table of integrals and a calculator.

Top 10 skills:

1. Solve linear first-order ODEs/initial value problems. (**Memorize the formulas!!**)
2. Solve nonlinear first-order ODEs/initial value problems if they are separable or exact.
   - If problem is separable, separate variables and integrate to get implicit solution.
   - Given an equation in the form $M(t,y) + N(t,y)y' = 0$, determine if it is exact, and if so, use anti-partial differentiation to determine a function $H(t,y)$ satisfying $\frac{\partial H}{\partial t} = M$ and $\frac{\partial H}{\partial t} = N$. Implicit solution is then $H(t,y) = C$.
   - If solving an initial value problem, determine the constant by plugging the initial value into the *implicit* solution.
   - Convert to explicit solution if possible.
3. Euler's method: Given a first order initial value problem, perform a couple steps of Euler's method.
4. Applications of first-order ODEs: Population growth and decay, Cooling problems, Free-Falling Bodies.
5. Solve an initial value problem involving a nonhomogeneous 2nd order linear differential equation with constant coefficients.
   - Form characteristic equation and determine its roots.
   - Construct general solution to homogeneous equation by using roots of characteristic equation (3 cases: distinct real roots, complex conjugate roots, repeated real roots).
   - Use method of undetermined coefficients to find particular solution to nonhomogeneous equation.
   - Plug in initial conditions ($y(0)$ and $y'(0)$) to resolve constants.
6. Modeling Spring-Mass Systems
   - Hooke's Law
   - damping coefficient
   - Behavior: (3 cases: overdamped, critically damped, underdamped).
   - Vibrations
   - period, frequency
   - Forced Mechanical Vibrations
7. Use the Wronskian to determine whether two given solutions (to a 2nd order linear ODE, or a 2×2 system of linear ODEs) form a fundamental set.
8. Find the general solution to a homogeneous system of 2 or 3 linear differential equations with constant coefficients (for the cases where there are no repeated eigenvalues).
   - Construct the general solution from the eigenvalues and eigenvectors of the coefficient matrix (for distinct real or complex conjugate eigenvalues).

9. Find the general solution to a 2-dimensional homogeneous first order linear system with constant coefficients. Know what to do for each of the following cases:
    - 2 distinct real eigenvalues.
    - Repeated eigenvalue, with 2 linearly independent eigenvectors.
    - Repeated eigenvalue, with only one eigenvector.
    - Complex conjugate eigenvalues.
10. Solve a nonhomogeneous 2-dimensional linear system with constant coefficients, of the form $\mathbf{X}' = A\mathbf{X} + \mathbf{F}(t)$, using the method of undetermined coefficients to find a particular solution.

Basic Skills/Terminology

- Classification of differential equations: order, linear vs. nonlinear, homogeneous vs. nonhomogeneous, constant coefficients vs. non-constant coefficients.
- Implicit vs. explicit solutions (on an interval).
- Differentiation/integration techniques.
- Linear combination, linear independence, determinant.
- Fundamental set of solutions, Wronskian (both for 2nd or higher order ODEs, and for linear systems).
- Characteristic equation.
- Euler's formula.
- Superposition principle.
- Systems of first order linear differential equations.
- Matrix functions.
    - Equality, addition, scalar multiplication, matrix multiplication, inversion.
    - Limits, derivatives and antiderivatives of matrix functions.
    - Representing linear systems in matrix terms.

A-level Material To get an "A", you are responsible for all of the material on all of the study guides and the homework. You should also be able to demonstrate understanding of the theorems and the thinking behind the various methods. The following is a partial list of topics that we covered, which are not mentioned above:

1. Existence and Uniqueness Theorems for nonlinear ODEs and linear ODEs.
2. Direction Fields
3. Phase Planes (classification of equilibrium solutions).
4. Accuracy of Euler's Method. Error Analysis of Euler's Method.
5. Reduction of order.
6. Qualitative behavior of solutions to 2nd order equations: Amplitude and phase.
    - Writing the solution using a single cosine function. $y(t) = Re^{\alpha t}\cos(\beta t - \delta)$.