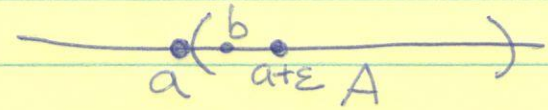
Quiz #2

$A \subset \mathbb{R}$, $a = \inf A \notin A$

prove $\exists (a_n) \subset A$ s/t $a_n \to a$.

pf: Since $a = \inf A$, $\forall \varepsilon > 0$, $\exists b \in A$ s/t $b < a + \varepsilon$



note: if b did not exist Then a is not The greatest lower bound of A.

So, choose $a_n \in A$ s/t $a_n < a + \frac{1}{n}$

Then $a \leq a_n < a + \frac{1}{n}$

This could be < since $a \notin A$

so $a_n \to a$

note: The proof would be The same even if $a \in A$.

Section 10

6a $|s_{n+1} - s_n| < 2^{-n}$

let $m > n$, Then

$$|s_m - s_n| = |s_m - s_{m-1} + s_{m-1} - s_{m-2} + s_{m-2} \cdots + s_{n+1} - s_n|$$

$$\leq \underbrace{|s_m - s_{m-1}|}_{< \frac{1}{2^{m-1}}} + \underbrace{|s_{m-1} - s_{m-2}|}_{< \frac{1}{2^{m-2}}} + \cdots + \underbrace{|s_{n+1} - s_n|}_{< \frac{1}{2^n}}$$

$$< \sum_{i=n}^{m-1} \frac{1}{2^i} < \sum_{i=n}^{\infty} \frac{1}{2^i} = \frac{1}{2^{n-1}}$$

$\therefore$ if $\frac{1}{2^N} < \varepsilon$ Then

$$n, m > N \implies |s_m - s_n| < \frac{1}{2^{n-1}} \leq \frac{1}{2^N} < \varepsilon$$

Since ε arbitrary, (s_n) is Cauchy sequence.

(8) $(s_n) \nearrow$ (increasing sequence)

$\sigma_n = \frac{s_1 + s_2 + \cdots + s_n}{n}$ "average" of $\{s_1, \cdots, s$

Show $(\sigma_n) \nearrow$

Pf: first, note That

$$\sigma_n = \frac{s_1 + s_2 + \cdots + s_n}{n} \leq \frac{s_n + s_n + \cdots + s_n}{n} = s_n$$

also, $\sigma_{n+1} = \frac{s_1 + \cdots + s_n + s_{n+1}}{n+1} = \frac{s_1 + \cdots + s_n}{n} \cdot \frac{n}{n+1} + \frac{s_n}{n+}$

$$= \sigma_n \cdot \frac{n}{n+1} + s_{n+1} \cdot \frac{1}{n+1}$$

$$\therefore \sigma_{n+1} - \sigma_n = \sigma_n \cdot \frac{n}{n+1} + s_{n+1} \cdot \frac{1}{n+1} - \sigma_n$$

$$\cancel{= \sigma_n \cdot \frac{1}{n+1}} = \frac{s_{n+1} - \sigma_n}{n+1} \geq \frac{s_{n+1} - s_n}{n+1} \geq 0$$