MATH 4320
midterm exam
Due March 20, 2008

1. Suppose $f_n \to f$ uniformly, and $\forall n,\ g_{ni} \to f_n$ uniformly.
   (a) Prove or find a counterexample: $g_{nn} \to f$.
   (b) Now, suppose that $\forall n,\ \forall i,\ |g_{ni} - f_n| < \frac{1}{i}$. Prove that $g_{nn} \to f$ uniformly.

2. Prove that if $0 < a < b < \infty$ then

$$\int_a^b \frac{dx}{1 - e^{-x}} = b - a\ +\ \sum_{n=1}^{\infty} \frac{e^{-na} - e^{-nb}}{n}.$$

   HINT: Use the geometric sum formula.

3. Let $F : \Re^n \to \Re^m$ be continuous, and for $A \subset \Re^n$ define $F(A) = \{F(x)\ :\ x \in A\}$.
   (a) Prove or find a counterexample: If A is open in $\Re^n$ then $F(A)$ is open in $\Re^m$.
   (b) Suppose that $\forall \epsilon,\ \forall x,\quad F(B_\epsilon(x)) \subset B_\epsilon(F(x))$. Prove that F is uniformly continuous.