Stochastic Processes (Math 6380)
final exam
Due Wednesday, December 15, 2010

Answer the following problems as best you can. For an "A" on a problem, you need a correct analytical solution. On problems 1 and 3, if you cannot find an "A" or "B" solution analytically, you can still get a "B" on the problem by finding the solution by simulation.

1. Let $\{X_0, X_1, \ldots\}$ be a Markov chain with pransition matrix,

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

Let $X_0 = 1$ and define $T = \min\{n \ : \ X_n = 3\}$.

   (a) Find $E(T)$ and $Var(T)$.

   (b) Find $\lim_{n\to\infty} P(X_n = i), \ i = 1, 2, 3$.

2. Let $N(t)$ be a Poisson process with rate λ. Let $0 < s < t$ and $j \leq k$. Find an expression for

   (a) $P(N(t) = k \mid N(s) = j)$

   (b) $P(N(s) = j \mid N(t) = k)$

3. Let W_t be standard Brownian motion, and define

$$M = \sup\{W_t \ : \ t \in [0,1]\}, \qquad \text{and} \qquad m = \inf\{W_t \ : \ t \in [0,1]\}.$$

Find $E(M - m)$. BONUS: Find $Var(M - m)$.