

Stochastic Processes (Math 6380)

take home exam

Due Wednesday, October 20, 2010

Try to answer all the questions. If you don't know how to do a problem, please explain as best you can where you are stuck instead of writing down things you know are wrong!

1. In the "shoe shine shop" model from class, find the expected time until the first customer is turned away. What fraction of the arrivals to the shop are turned away?

2. Let X_t be a Poisson process with rate λ , and let Y_k be the time of the k th event.

(a) Write an integral expression for $P(Y_2 \leq t)$. HINT: condition on Y_1 .

(b) Show that

$$P(Y_k \leq t) = \sum_{\ell=k}^{\infty} \frac{(\lambda t)^\ell e^{-\lambda t}}{\ell!}.$$

HINT: Argue that $P(Y_k \leq t) = P(X_t \geq k)$.

3. Let $X_1(t)$ and $X_2(t)$ be Poisson processes with rates λ_1 and λ_2 , and Let $Z(t) = X_1(t) - X_2(t)$.

(a) What is the infinitesimal generator for $Z(t)$?

(b) Assume $\lambda_1 > \lambda_2$. Let $T = \inf\{t : Z(t) = 1\}$. Find $E(T)$.

(c) BONUS: Find $Var(T)$. (This could be tricky!)