Factoring involves a certain amount of trial and error, which can become frustrating, especially when the leading coefficient is not 1. You might want to try a rather neat scheme that will greatly reduce the number of candidates.

We’ll demonstrate the method for the polynomial

\[ 4x^2 + 11x + 6 \]  

Using the leading coefficient of 4 we write the pair of incomplete factors

\[ (4x \quad ) (4x \quad ) \]  

Next, multiply the coefficient of \( x^2 \) and the constant term in (1) to produce \( 4 \cdot 6 = 24 \). Now find two integers whose product is 24 and whose sum is 11, the coefficient of the middle term of (1). It’s clear that 8 and 3 will do nicely, so we write

\[ (4x+8)(4x+3) \]  

Finally, within each parenthesis in (3) discard any common divisor. Thus \( 4x+8 \) reduces to \( (x+2) \) and we write

\[ (x + 2)(4x + 3) \]  

which is the factorization of \( 4x^2 + 11x + 6 \).

Will the method always work? Yes—if you first remove all common factors in the original polynomial. That is, you must first write

\[ 6x^2 + 15x + 6 = 3(2x^2 + 5x + 2) \]

and apply the method to the polynomial \( 2x^2 + 5x + 2 \).

(For a proof that the method works, see M. A. Autrie and J. D. Austin, “A Novel Way to Factor Quadratic Polynomials.”, The Mathematics Teacher 72 no. 2[1979].)

We’ll use the polynomial \( 2x^2 – x – 6 \) of Example 7 to demonstrate the method when some of the coefficients are negative.

<table>
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<th>Factoring ( ax^2 + bx + c )</th>
<th>Example: ( 6x^2 + 7x - 3 )</th>
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<tr>
<td><strong>Step 1.</strong> Using the leading coefficient of ( a ) we write the pair of incomplete factors</td>
<td><strong>Step 1.</strong> The lead coefficient is 6, so we write</td>
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<td><strong>Step 2.</strong> Multiply ( a ) and ( c ), the coefficients of ( x^2 ) and the constant term.</td>
<td><strong>Step 2.</strong> ( a \cdot c = (6)(-3) = -18 )</td>
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<td><strong>Step 3.</strong> Find integers whose product is ( a \cdot c ) and the constant term</td>
<td><strong>Step 3.</strong> Two integers whose product is (-18) and whose sum is ( 7 ) are ( 9 ) and ( -2 ). Then we write ( (6x + 9)(6x - 2) )</td>
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<td><strong>Step 4.</strong> Discard any common factor within each parenthesis in Step 3. The result is the desired factorization.</td>
<td><strong>Step 4.</strong> Reducing ( 6x + 9 ) to ( (2x + 3) ) and ( 6x - 2 ) to ( (3x - 1) ) we have ( 6x^2 + 7x - 3 = (2x+3)(3x-1) )</td>
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