

Complexity Results for Rainbow Matchings

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Abstract

A rainbow matching in an edge-colored graph is a matching whose edges have distinct colors. We address the complexity issue of the following problem, MAX RAINBOW MATCHING: Given an edge-colored graph G , how large is the largest rainbow matching in G ? We present several sharp contrasts in the complexity of this problem.

We show, among others, that

- MAX RAINBOW MATCHING can be approximated by a polynomial algorithm with approximation ratio $2/3 - \varepsilon$.
- MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored linear forests without a 5-vertex path, and is solvable in polynomial time for edge-colored forests without a 4-vertex path.
- MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored trees without an 8-vertex path, and is solvable in polynomial time for edge-colored trees without a 7-vertex path.
- MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored paths.

These results provide a dichotomy theorem for the complexity of the problem on forests and trees in terms of forbidding paths. The latter is somewhat surprising, since, to the best of our knowledge, no (unweighted) graph problem prior to our result is known to be NP-hard for simple paths.

We also address the parameterized complexity of the problem.

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1 Introduction and Results

Given a graph $G = (E(G), V(G))$, an edge coloring is a function $\phi : E(G) \rightarrow \mathcal{C}$ mapping each edge $e \in E(G)$ to a color $\phi(e) \in \mathcal{C}$; ϕ is a *proper* edge-coloring if, for all distinct edges e and e' , $\phi(e) \neq \phi(e')$ whenever e and e' have an endvertex in common. A (properly) edge-colored graph (G, ϕ) is a pair of a graph together with a (proper) edge coloring. A *rainbow subgraph* of an edge-colored graph is a subgraph whose edges have distinct colors. Rainbow subgraphs appear frequently in the literature, for a recent survey we point to [14].

In this paper we are concerned with *rainbow matchings*, i.e., matchings whose edges have distinct colors. One motivation to look at rainbow matchings is Ryser's famous conjecture



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from [21], which states that every Latin square of odd order contains a Latin transversal. Equivalently, the conjecture says that every proper edge coloring of the complete bipartite graph $K_{2n+1,2n+1}$ with $2n + 1$ colors contains a rainbow matching with $2n + 1$ edges.

One often asks for the size of the largest rainbow matching in an edge-colored graph with certain restrictions (see, e.g., [4, 15, 23]). In this paper, we consider the complexity of this problem. In particular, we consider the complexity of the following two problems, and we will restrict them to certain graph classes and edge colorings.

RAINBOW MATCHING

Instance: Graph G with an edge-coloring and an integer k

Question: Does G have a rainbow matching with at least k edges?

RAINBOW MATCHING is also called MULTIPLE CHOICE MATCHING in [8, Problem GT55]. The optimization version of the decision problem RAINBOW MATCHING is:

MAX RAINBOW MATCHING

Instance: Graph G with an edge-coloring

Output: A largest rainbow matching in G .

Only the following complexity result for RAINBOW MATCHING is known. Note that the considered graphs in the proof are not properly edge colored. When restricted to properly edge-colored graphs, no complexity result is known prior to this work.

► **Theorem 1** ([13]). *RAINBOW MATCHING is NP-complete, even when restricted to edge-colored bipartite graphs.*

In this paper, we analyze classes of graphs for which MAX RAINBOW MATCHING can be solved and thus RAINBOW MATCHING can be decided in polynomial time, or is NP-hard. Our results are:

- There is a polynomial-time $(2/3 - \varepsilon)$ -approximation algorithm for MAX RAINBOW MATCHING for every $\varepsilon > 0$.
- MAX RAINBOW MATCHING is APX-complete, and thus RAINBOW MATCHING is NP-complete, even for very restricted graphs classes such as
 - edge-colored complete graphs,
 - properly edge-colored paths,
 - properly edge-colored P_8 -free trees in which every color is used at most twice,
 - properly edge-colored P_5 -free linear forests in which every color is used at most twice,
 - properly edge-colored P_4 -free bipartite graphs in which every color is used at most twice.

These results significantly improve Theorem 1. We also provide an inapproximability bound for each of the listed graph classes.

- MAX RAINBOW MATCHING is solvable in time $O(m^{3/2})$ for m -edge graphs without P_4 (induced or not); in particular for P_4 -free forests.
- MAX RAINBOW MATCHING is polynomially solvable for P_7 -free forests with bounded number of components; in particular for P_7 -free trees.
- MAX RAINBOW MATCHING is fixed parameter tractable for P_5 -free forests, when parameterized by the number of the components.

The next section contains some relevant notation and definitions. Section 3 deals with approximability and inapproximability results, Section 4 discusses some polynomially solvable cases, and Section 5 addresses the parameterized complexity. We conclude the paper in Section 6 with some open problems.

2 Definitions and Preliminaries

We consider only finite, simple, and undirected graphs. For a graph G , the vertex set is denoted $V(G)$ and the edge set is denoted $E(G)$. An edge xy of a graph G is a *bridge* if $G - \{x, y\}$ has more components than G . If G does not contain an induced subgraph isomorphic to another graph F , then G is F -free.

For $\ell \geq 1$, let P_ℓ denote a chordless path with ℓ vertices and $\ell - 1$ edges, and for $\ell \geq 3$, let C_ℓ denote a chordless cycle with ℓ vertices and ℓ edges. A *triangle* is a C_3 . For $p, q \geq 1$, $K_{p,q}$ denotes the complete bipartite graph with p vertices of one color class and q vertices of the second color class; a *star* is a $K_{1,q}$. A complete graph with p vertices is denoted by K_p ; $K_p - e$ is the graph obtained from K_p by deleting one edge. An r -regular graph is one in which each vertex has degree exactly r . A forest in which each component is a path is a *linear forest*.

The *line graph* $L(G)$ of a graph G has vertex set $E(G)$, and two vertices in $L(G)$ are adjacent if the corresponding edges in G are incident. By definition, every matching in G corresponds to an independent set in $L(G)$ of the same size, and vice versa. One of the main tools we use in discussing rainbow matchings is the following concept that generalizes line graphs naturally:

► **Definition 2.** The *color-line graph* $CL(G)$ of an edge-colored graph G has vertex set $E(G)$, and two vertices in $CL(G)$ are adjacent if the corresponding edges in G are incident or have the same color.

Notice that, given an edge-colored graph G , $CL(G)$ can be constructed in time $O(|E(G)|^2)$ in an obvious way. We will make use of further facts about color-line graphs below that can be verified by definition.

► **Lemma 3.** *Let G be an edge-colored graph. Then*

- (i) $CL(G)$ is $K_{1,4}$ -free.
- (ii) $CL(G)$ is $(K_7 - e)$ -free, provided G is properly edge-colored.
- (iii) Every rainbow matching in G corresponds to an independent set in $CL(G)$ of the same size, and vice versa.

Lemma 3 allows us to use results about independent sets to obtain results on rainbow matchings. This way, we will relate MAX RAINBOW MATCHING to the following two problems, which are very well studied in the literature.

MIS (MAXIMUM INDEPENDENT SET)

Instance: A graph G .

Output: A maximum independent set in G .

3-MIS (MAXIMUM INDEPENDENT SET IN 3-REGULAR GRAPHS)

Instance: A 3-regular graph G .

Output: A maximum independent set in G .

In the present paper, a polynomial-time algorithm A with approximation ratio α , $0 < \alpha < 1$, for a (maximization) problem is one that, for all problem instances I , $A(I) \geq \alpha \cdot \text{opt}(I)$, where $A(I)$ is the objective value of the solution found by A and $\text{opt}(I)$ is the objective value of an optimal solution. A problem is said to be in APX (for approximable) if it admits an algorithm with a constant approximation ratio. A problem in APX is called APX-complete if all other problems in APX can be L -reduced (cf. [19]) to it. It is known that 3-MIS is APX-complete (see [1]). Thus, if 3-MIS is L -reducible to a problem in APX, then this problem is also APX-complete. All reductions in this paper are L -reductions.

3 Approximability and Hardness

We first show that MAX RAINBOW MATCHING is in APX, by reducing to MIS on $K_{1,4}$ -free graphs. The following theorem is due to Hurkens and Schrijver [12]; see also [11].

► **Theorem 4** ([12]). *For every $\varepsilon > 0$ and $p \geq 3$, MIS for $K_{1,p+1}$ -free graphs can be approximated by a polynomial algorithm with approximation ratio $2/p - \varepsilon$.*

► **Theorem 5.** *For every $\varepsilon > 0$, MAX RAINBOW MATCHING can be approximated by a polynomial algorithm with approximation ratio $2/3 - \varepsilon$.*

Proof. This follows from Lemma 3 and Theorem 4 with $p = 3$. ◀

On the other hand, we show that MAX RAINBOW MATCHING is APX-complete, and thus RAINBOW MATCHING is NP-complete, even when restricted to very simple graph classes, and we give some inapproximability bounds for MAX RAINBOW MATCHING. We will reduce MAX RAINBOW MATCHING on these graph classes to 3-MIS, and use the following theorem by Berman and Karpinski [2], where the second part of the statement does not appear in the original statement, but follows directly from their proof.

► **Theorem 6** ([2]). *For any $\varepsilon \in (0, 1/2)$, it is NP-hard to decide whether an instance of 3-MIS with $284n$ nodes has the maximum size of an independent set above $(140 - \varepsilon)n$ or below $(139 + \varepsilon)n$. The statement remains true if we restrict ourselves to the class of bridgeless triangle-free 3-regular graphs.*

► **Theorem 7.** *MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored 2-regular graphs in which every color is used exactly twice. Unless $P=NP$, no polynomial algorithm can guarantee an approximation ratio greater than $\frac{139}{140}$.*

Proof. Let G be a bridgeless triangle-free 3-regular graph on $284n$ vertices. By a classical theorem of Petersen [20], G contains a perfect matching M . Then, $G - M$ is triangle-free and 2-regular, and thus the line graph of a triangle-free and 2-regular graph H on $284n$ vertices. Now it is easy to color the edges of H in such a way that every color is used exactly twice, and $G = CL(H)$. From Theorem 6, it follows that it is NP-hard to decide if the maximal size of a rainbow matching in H is above $(140 - \varepsilon)n$ or below $(139 + \varepsilon)n$. ◀

► **Corollary 8.** *MAX RAINBOW MATCHING is APX-complete, even when restricted to complete graphs. Unless $P=NP$, no polynomial algorithm can guarantee an approximation ratio greater than $\frac{139}{140}$.*

Proof. Use the same graph H from the previous proof, add two new vertices, and add all missing edges, all colored with the same new color, to get an edge-colored complete graph H' on $284n + 2$ vertices. Then, it is NP-hard to decide if the maximal size of a rainbow matching in H' is above $(140 - \varepsilon)n + 1$ or below $(139 + \varepsilon)n + 1$. ◀

► **Theorem 9.** *MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored paths. Unless $P=NP$, no polynomial algorithm can guarantee an approximation ratio greater than $\frac{210}{211}$.*

Proof. Again, start with the 2-regular graph H from the proof of Theorem 7. For a cycle $C \subseteq H$, create a path P_C as follows. Cut the cycle at a vertex v to get a path of the same length as C , with the two end vertices corresponding to the original vertex v . Now add an extra edge to each of the two ends, and color this edge with the color v —a color not used

anywhere else in H . The maximum rainbow matching in this new graph H' is exactly one greater than the maximum rainbow matching in H . To see this, take a maximum rainbow matching in H , and notice that it can contain at most one edge incident to v . Thus, in H' we can add one of the two edges colored v to this matching to get a greater rainbow matching. On the other hand, every rainbow matching in H contains at most two edges incident to the two copies of v , and at most one of them is not in H . Thus, deleting one edge from a rainbow matching in H' yields a rainbow matching in H .

Now repeat this process for every cycle in H to get a linear forest L with c components, say, and $284n + 2c$ edges. Similarly to above, it is NP-hard to decide if the maximal size of a rainbow matching in L is above $(140 - \varepsilon)n + c$ or below $(139 + \varepsilon)n + c$. We now connect all paths in L with $c - 1$ extra edges colored with a new color 1 to one long path, and add a path on 5 edges colored 1, 2, 1, 2, 1 (where 2 is a new color) to one end to get a path P on $284n + 3c + 4$ edges. The size of a maximum rainbow matching in P is exactly 2 larger than in L , so it is NP-hard to decide if the maximal size of a rainbow matching in P is above $(140 - \varepsilon)n + c + 2$ or below $(139 + \varepsilon)n + c + 2$. As H does not contain any triangles, we have $c \leq 71n$, and the theorem follows. ◀

► **Theorem 10.** MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored P_5 -free linear forests in which every color is used at most twice. Unless $P=NP$, no polynomial algorithm can guarantee an approximation ratio greater than $\frac{423}{424}$.

Proof. We again start with the 2-regular graph H from the proof of Theorem 7. Now construct a linear forest L consisting of $284n = |E(H)|$ paths of length 3, where every edge in H corresponds to one path component in L . For an edge $vw \in E(H)$ with color $\phi(vw)$, color the three edges of the corresponding path with the colors v , $\phi(vw)$ and w in this order. We claim that a maximum rainbow matching in L is exactly $284n$ greater than a maximum rainbow matching in H . Note that this claim implies the theorem. To see the claim, consider first a rainbow matching M in H . Note that for every vertex $v \in V(H)$, M can contain at most one edge incident to v , so in L we can add one of the two edges labeled v to the matching induced by M . This can be done for every vertex in $V(H)$, so the largest rainbow matching in L is at least $284n$ larger than M . On the other hand, every rainbow matching M' in L contains at most two edges either colored v or incident to an edge colored v , and at most one of them is colored v . Thus, by deleting at most $284n$ edges from M' we can create a rainbow matching in H . Therefore, it is NP-hard to decide if the maximal size of a rainbow matching in L is above $(140 + 284 - \varepsilon)n$ or below $(139 + 284 + \varepsilon)n$. ◀

► **Theorem 11.** MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored P_4 -free bipartite graphs in which every color is used at most twice. Unless $P=NP$, no polynomial algorithm can guarantee an approximation ratio greater than $\frac{423}{424}$.

Proof. Take the linear forest L from the proof of Theorem 10, add an edge to every P_4 to make it a C_4 , and color the new edge with the same color as the middle edge of the P_4 . This graph G is P_4 -free, and every rainbow matching in G corresponds to a rainbow matching in L of the same size. ◀

► **Theorem 12.** MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored P_6 -free linear forests in which every color is used at most twice. Unless $P=NP$, no polynomial algorithm can guarantee an approximation ratio greater than $\frac{1689}{1694}$.

Proof. Similarly to above, we start with the 2-regular graph H from the proof of Theorem 7, and transform it into a linear forest L similarly to the last two proofs. This time, we try to

produce paths of length 4 by splitting the cycles in H only at every other vertex if possible. As H may contain odd cycles, we have to use one path of length only 3 for every one of the o odd cycles in H . Thus, L has exactly $(284n + o)/2$ component paths, and a maximum rainbow matching in L is exactly $(284n + o)/2$ greater than a maximum rainbow matching in H . As H is triangle-free, we know that $o \leq 284n/5$, so it is NP-hard to decide if the maximal size of a rainbow matching in L is above $(140 + 0.7 \times 284 - \varepsilon)n$ or below $(139 + 0.7 \times 284 + \varepsilon)n$. ◀

► **Theorem 13.** *MAX RAINBOW MATCHING is APX-complete, even when restricted to properly edge-colored P_8 -free trees in which every color is used at most twice. Unless $P=NP$, no polynomial algorithm can guarantee an approximation ratio greater than $\frac{1689}{1694}$.*

Proof. Start with an edge-colored linear forest L as in the proof of Theorem 12. Add an extra vertex v , and connect it to a central vertex (i.e., a vertex with maximum distance to the ends) in every path in L . Further, add one pending edge to v . Color the added edges with colors not appearing on L . Then the resulting tree T is P_8 -free, and a maximum rainbow matching in T is exactly one edge larger than a maximum rainbow matching in L . ◀

All these theorems imply the following complexity result on RAINBOW MATCHING.

► **Corollary 14.** *RAINBOW MATCHING is NP-complete, even when restricted to one of the following classes of edge-colored graphs.*

1. Complete graphs.
2. Properly edge-colored paths.
3. Properly edge-colored P_5 -free linear forests in which every color is used at most twice.
4. Properly edge-colored P_4 -free bipartite graphs in which every color is used at most twice.
5. Properly edge-colored P_8 -free trees in which every color is used at most twice.

4 Polynomial-time Solvable Cases

In contrast to Theorem 11, saying that MAX RAINBOW MATCHING is hard even for P_4 -free bipartite graphs, we have:

► **Theorem 15.** *In every graph G which does not contain P_4 as a not necessarily induced subgraph, MAX RAINBOW MATCHING is solvable in time $O(m^{3/2})$, where m is the number of edges in G .*

Proof. As G does not contain a P_4 , every component of G is either a star or a triangle. Now construct a bipartite graph H with partite sets being the components of G and the colors used in G . The graph H has an edge between a component and a color if in G , the color appears in the component. The graph H has at most as many edges as G , and rainbow matchings in G correspond to matchings in H of the same size. As we can find a maximum matching in the bipartite graph in time $O(m^{3/2})$ ([10, 17, 22]), the same is true for G . ◀

Since in a forest, every P_4 is an induced subgraph, we have the following positive result complementing Theorem 10:

► **Corollary 16.** *In every P_4 -free forest F , MAX RAINBOW MATCHING is solvable in time $O(n^{3/2})$, where n is the number of vertices in F .*

In contrast to Theorem 13, saying that MAX RAINBOW MATCHING is hard even for P_8 -free trees, we have:

► **Theorem 17.** *In every tree T which does not contain P_7 as a subgraph, MAX RAINBOW MATCHING is solvable in time $O(n^{7/2})$, where n is the number of vertices in T .*

Proof. As T is P_7 -free, we can find an edge xy in T such that $G - \{x, y\}$ is a forest consisting of stars; all we have to do is to pick the two most central vertices in a longest path in T and note that every vertex of T must have distance at most 2 to $\{x, y\}$. Every matching M can contain at most 2 edges incident to $\{x, y\}$. Once we have decided on these at most two edges (less than n^2 choices), we are left with the task of finding a rainbow matching in a P_4 -free forest, which can be done in time $O(n^{3/2})$ by Theorem 15. This gives a total time of $O(n^{7/2})$. ◀

The following theorem describes a more general setting of Theorem 17:

► **Theorem 18.** *In every forest F which does not contain P_7 as a subgraph, MAX RAINBOW MATCHING is solvable in time $O(\frac{1}{2^k k^{2k}} n^{(4k+3)/2})$, where n is the number of vertices in F and k is the number of components in F .*

Proof. As in proof of Theorem 17, find a central edge $x_T y_T$ in every component $T \subseteq F$, such that $\bigcup (T - \{x_T, y_T\})$ is a forest consisting of stars. Once we have decided on the at most $2k$ edges in a matching incident to the $x_T y_T$ —a total of at most $\binom{n/k}{2}^k < \frac{n^{2k}}{2^k k^{2k}}$ choices—we are left with the task of finding a rainbow matching in a P_4 -free forest, which can be done in $O(n^{3/2})$ by Theorem 15. This gives a total time of $O(\frac{1}{2^k k^{2k}} n^{(4k+3)/2})$. ◀

► **Corollary 19.** *MAX RAINBOW MATCHING is solvable in polynomial time for P_7 -free forests with bounded number of components.*

5 Fixed Parameter Aspects

An approach to deal with NP-hard problems is to fix a parameter when solving the problems. A problem parameterized by k is *fixed parameter tractable*, *fpt* for short, if it can be solved in time $f(k) \cdot n^{O(1)}$, or, equivalently, in time $O(n^{O(1)} + f(k))$, where $f(k)$ is a computable function, depending only on the parameter k . For an introduction to parameterized complexity theory, see for instance [5, 7, 18].

Observe that MAX RAINBOW MATCHING is fpt, when parameterized by the size of the problem solution. In case of properly edge-colored inputs, this is a consequence of Lemma 3, and of the fact that MIS is fpt for $(K_7 - e)$ -free graphs ([3]). In the general case, rainbow matchings can be seen as matching (set packing) in certain 3-uniform hypergraphs, hence MAX RAINBOW MATCHING is fpt by a result of Fellows et al. [6].

Recall that MAX RAINBOW MATCHING is already hard for P_8 -free trees. In view of Theorem 18, we now consider MAX RAINBOW MATCHING for P_7 -free forests, parametrized by k , the number of components in the inputs. Formally, we want to address the following parameterized problem:

k -FOREST RAINBOW MATCHING

Instance: A P_7 -free forest F with k components containing a P_4 .

Parameter: k .

Output: A maximum rainbow matching in F .

Theorem 20 below shows that k -FOREST RAINBOW MATCHING is fpt for P_5 -free forests.

► **Theorem 20.** *In every forest F which does not contain P_5 as a subgraph, MAX RAINBOW MATCHING is solvable in time $O(n + 2^k k^3)$, where n is the number of vertices in F and k is the number of components in F containing a P_4 .*

Proof. Let $E' \subset E(F)$ be the set of edges between vertices of degree greater than 1, and observe that $|E'| \leq k$. Then, $F' = F - E'$ is a forest consisting of at most $2k$ stars. Delete edges in F' until each star is rainbow colored and has at most $2k$ edges. Notice that this does not change the size of a maximum rainbow matching. Call this new graph F'' , and observe that $|E(F'')| \leq 4k^2$.

Now for every rainbow choice of edges in E' , we can solve MAX RAINBOW MATCHING on a subgraph of F'' . There are at most 2^k rainbow choices in E' , and solving MAX RAINBOW MATCHING on F'' takes time $O(k^3)$ as in Theorem 15. Creating F'' from F takes time $O(n)$, so the result follows. ◀

We do not know if k -FOREST RAINBOW MATCHING is fpt for P_6 -free forests or $W[1]$ -hard. Note that k -FOREST RAINBOW MATCHING is in XP by Theorem 18.

► **Remark.** If k -FOREST RAINBOW MATCHING for P_6 -free forests is in $W[i]$, then so is k -FOREST RAINBOW MATCHING for P_7 -free forests.

Proof. Let F be a P_7 -free forest with k components containing a P_4 . For every tree $T \subseteq F$ containing a P_6 , find the central edge $x_T y_T$. Once you choose one of the 2^k possibilities to include these central edges in a rainbow matching, delete the not-chosen edges, and delete the chosen edges together with their neighborhood, the remaining graph contains at most $2k$ components containing a P_4 , and no components containing a P_6 . ◀

6 Concluding Remarks

We have shown that it is NP-hard to approximate MAX RAINBOW MATCHING within certain ratio bounds for very restricted graph classes. Implicit in our results is the following dichotomy theorem for forests and trees in terms of forbidding paths: RAINBOW MATCHING is NP-complete for P_5 -free forests (P_8 -free trees), and is polynomially solvable for P_4 -free forests (P_7 -free trees).

We have also proved that k -FOREST RAINBOW MATCHING is fixed parameter tractable for P_5 -free forests. What can we find out about the parameterized complexity in the only open case of P_6 -free (and equivalently, P_7 -free) forests?

Another open problem of independent interest is the computational complexity of recognizing color-line graphs: Given an edge-colored graph G , does there exist an edge-colored graph H such that $G = CL(H)$? Note that it is well-known that line graphs can be recognized in linear time.

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