For office use only
T1 ____________
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Team Control Number
815

Problem Chosen
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For office use only
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2005 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

The ozone layer is a highly valuable, non-renewable resource upon which all forms of life on Earth rely. Since organic organisms are extremely sensitive to the Sun's ultra-violet radiation, life as we know it would not exist without the protection of the ozone layer.

Using data on past emissions of ozone depleting substances, we generated two models that estimate future amounts of ozone depletion. We used the first to forecast how current ODS consumption rates would affect future ozone depletion without any restrictions in place. Our next model hypothesized the long term effects of substituting more environmentally friendly HCFCs for the ozone-depleting CFC and Halon chemicals still presently in use.

The net forecasted effect of this substitution is a decrease in ozone depletion from 4.48 billion kg of O_3 to 445.19 million kg over the next fifteen years. We predict that this reduction will lead to 393 fewer deaths a year due to skin cancer by 2020. In addition, the economic ramifications of replacing ozone depleting substances with more expensive, less toxic material results in an overall cost increase of $535,542.00.

There is no existing technology that can effectively replace or mimic the function of our ozone layer. Currently, preventative measures are the only way of preserving it. Through careful administrative policies, however, we feel that it is possible to manage and protect our ozone from additional damage.

Because the dismantled O_3 molecules bond with Cl and Br more quickly than the free oxygen molecules, the ozone regeneration process is interrupted.[23] Over time, these interruptions have slowly depleted the stratospheric ozone layer and could become severe enough to permanently alter the way our atmosphere works.

Ozone depleting substances' destructive potential (ODP) is measured using atmospheric lifespan. An ODS has a high potential of depleting the ozone layer if it has a longer lifespan. A substance with a shorter atmospheric lifespan has a smaller potential of breaking up ozone, and thus a lower ODP.[11]

Ozone thickness can be represented in several ways. Most research is recorded in Dobson Units (DU), where 1 DU corresponds to 0.01 mm of pure ozone at standard temperature and pressure (STP).[30] Figure 3 illustrates the considerable amount of ozone loss in the Antarctic region over a period of ten years. Take into account that 500 DU is equivalent to a healthy amount of ozone, and anything less than 200 DU is considered an ozone hole.[23]

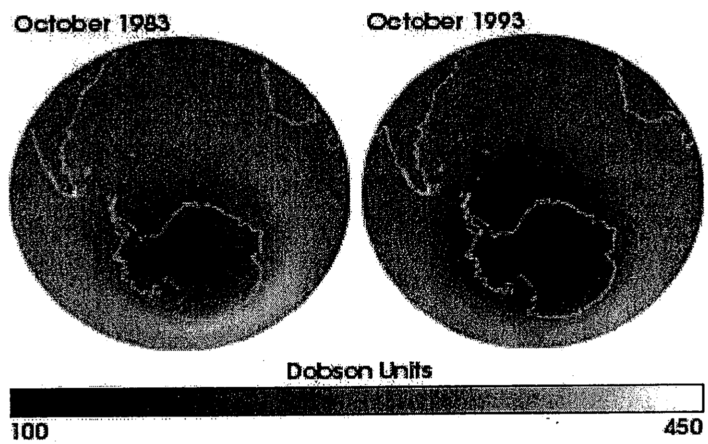


Figure 3 [28]
Antarctic Pole. Ozone levels measured by the Total Ozone Mapping Spectrometer (TOMS).

In 1985, scientists and researchers discovered two annually occurring holes above the arctic polar regions[20]; the damage had proven significant enough to trigger international concern for the health and wellbeing of the ozone layer. As a result of this concern, an international agreement was reached in 1987 entitled the Montreal Protocol. Its main goal was to execute regulations and bans on ODS use and production in all countries around the world. Designed by a group of environmentalists wanting to protect the ozone layer, its main proposition outlined a plan for the complete "phase out" of ODS use by the year 2000.[25]

Task 1: Long Term Depletion Model

Possible Modeling Methods

First, we contemplated several approaches to constructing an applicable model. Using Dobson Unit measurements taken around the globe, we originally planned to build a visual representation of the ozone layer which could be manipulated according to various conditions. We wanted to consider a representative longitudinal strip of the Earth, find points within that region at which ozone measurements were taken, and average them. We would then extend our calculations to account for ozone accumulation over the atmosphere. However, the data we were able to locate proved quite perplexing until we could find a key with which to decipher it. Even more frustrating was the realization that, after we had dealt with all the obstacles to understand what we found, there were simply not enough stations reporting data to compile our estimate in this manner. We abandoned this model as it seemed meaningless to compose it without sufficient applicable statistics.

Next, we considered modeling the ozone depletion as a function of population growth factors and forecasting future ozone attenuation with respect to that growth. In doing further research, however, we decided that other aspects (especially natural and environmental) weigh too significantly on the depletion rates to completely ignore them. Unfortunately, we were unable to find pertinent information – or effectively assume data – on these important factors, so we could not appropriately alter our model to account for changes in these variables.

Given more time and access to relevant data tables, expanded development of these concepts and their respective models would more than likely materialize.

Success At Last!

As we conducted our research, we continually faced the obstacle of very time-constrained data. We were disappointed to find the earliest recorded research on the ozone layer dated back only to the 1960s![24] Furthermore, it was difficult to locate concrete numerical figures from any historical time frame with which to formulate a mathematical representation. Eventually, however, we were able to recover some relevant data from 1986 forward and successfully manipulate it to create our models.

Also, the earliest records only address a very limited number of ozone depleting substances. Not until recently did scientists discover a multitude of other molecules that affect the ozone layer. Using the presently available data, we focused on the two most prominent ozone depleting substances, CFCs and Halons. We were able to compare their consumption and production in the past with the overall level of ozone depletion. Then we used this comparison to project long term damage of current and future ODS consumption.

Log-Linear Model

In order to build a feasible model to forecast the future amounts of ozone and ozone depleting substances we had to make certain assumptions about their behavior and the properties of the ozone itself:

- We assumed that both the ozone and ozone depleting substances are evenly distributed around the globe.
- In our model, the ozone depleting substances do all of their damage and get entirely washed away during the same year. Also, the rate of depletion per kilogram of ODS is constant over time and for all possible quantities of the substances.
- We weighted the ODP of the two major molecular families, CFCs and Halons, and factored them both into the model under the combined ODS category.
- We assumed that the relative industrial production and consumption rates of ODS have remained and will remain constant over time, leading to a constant proportion of the chemicals in the stratosphere.
- Also, due to data constraints, we considered 2000 to be the present (t = 0) and for the future period to start in 2001 (t = 1).

Linear Regression

After plotting the rate of change of ozone depleting substances over time, the data appears to be positively skewed and thus lends itself to a log transformation (Appendix T1). A simple regression line can now be used to create a log-linear forecasting model:

Let $\hat{y}_t = \mathrm{B}_0 + \mathrm{B}_1 \hat{x}_t$ be our log-linear model with $T - k = 13$ degrees of freedom and a variance of $\hat{\sigma}^2$, then:

$$\hat{\mathrm{B}}_1 = \frac{\sum_{i=-14}^{0}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=-14}^{0}(x_i - \bar{x})^2} = \frac{-53.651348}{280} = -.191612$$

$$\hat{\mathrm{B}}_0 = \bar{y} - \hat{\mathrm{B}}_1 \bar{x} = 5.847397 + (.191612)(1993) = 387.730028$$

$$\hat{\sigma}^2 = \sqrt{\frac{\sum_{i=-14}^{0}(\hat{e}_i)^2}{T-k}} = .755994$$

Therefore, our fitted log-linear model is $\hat{y}_t = 387.730028 - .191612\hat{x}_t$.

Table 1 -- Forecasted ODS Emissions Output

year	ODS per year (million kg)	year	ODS per year (million kg)	year	ODS per year (million kg)
1986	1072	1998	133	**2010**	**14**
1987	1046	1999	101	**2011**	**11**
1988	764	2000	94	**2012**	**9**
1989	664	**2001**	**77**	**2013**	**8**
1990	591	**2002**	**63**	**2014**	**6**
1991	506	**2003**	**52**	**2015**	**5**
1992	338	**2004**	**43**	**2016**	**4**
1993	254	**2005**	**36**	**2017**	**4**
1994	152	**2006**	**29**	**2018**	**3**
1995	159	**2007**	**24**	**2019**	**2**
1996	147	**2008**	**20**	**2020**	**2**
1997	147	**2009**	**17**	*bold indicates forecasted*	

Table 2 -- Forecasted Ozone Levels

year	Ozone (millions of kg)	year	Ozone (millions of kg)
2000	3,000,000	**2011**	**2,995,105.07**
2001	**2,999,028.45**	**2012**	**2,994,987.00**
2002	**2,998,226.33**	**2013**	**2,994,889.52**
2003	**2,997,564.05**	**2014**	**2,994,809.03**
2004	**2,997,017.09**	**2015**	**2,994,742.58**
2005	**2,996,565.64**	**2016**	**2,994,687.71**
2006	**2,996,192.90**	**2017**	**2,994,642.41**
2007	**2,995,885.15**	**2018**	**2,994,605.01**
2008	**2,995,631.06**	**2019**	**2,994,574.13**
2009	**2,995,421.28**	**2020**	**2,994,548.64**
2010	**2,995,248.07**	*bold indicates forecasted*	

Table 1 lists the ODS emissions from 1986 – 2020. Table 2 shows a slow decline in the amounts of ozone in the atmosphere from 2000 to 2020. As the amount of ODS emissions drops, the forecasted ozone levels fall as well when comparing Table 1 and Table 2 (also see Appendices C1 and C2). This does not mean that less ODS consumption equates to ozone depletion. The harmful molecules from ozone depleting substances are released into the atmosphere where they can wreak havoc on O_3 before they dissipate (which can take up to approximately 50 years).

Estimate of Forecasting Error

Now that we have a least squares line modeling the data up to the present time, we can use this model to forecast future ODS consumption. As with any forecasting model, further forecasts into the future are accompanied by larger variance in the forecast error. Consequently, we will check a forecasted point to assess the validity of our predictions.

Let $\hat{\sigma}^2(f)$ be the variance of the forecast error for the least squares estimator, then:

$$\hat{\sigma}^2(f) = \hat{\sigma}^2\left[1+\frac{1}{T}+\frac{(x_{10}-\bar{x})^2}{\sum_{i=-14}^{0}(x_i-\bar{x})^2}\right] = 0.75599375\left[1+\frac{1}{15}+\frac{(2010-1993)^2}{280}\right] = 1.586687$$

$$\hat{\sigma}(f) = \sqrt{\hat{\sigma}^2(f)} = 1.259638$$

Thus, the standard deviation of our forecast error is 1.2596 in the year 2010. A 90% confidence interval for the predicted value of $\hat{y}_{10} = 2.614$ and a $t_c = 1.771$ is:

$$P(\hat{y}_{10} - t_c\hat{\sigma}(f) \le y_{10} \le \hat{y}_{10} + t_c\hat{\sigma}(f)) = .90$$

$$P(2.614 - 1.771(1.259638) \le y_{10} \le 2.614 + (1.771)(1.259638)) = .90$$

$$P(.383181 \le y_{10} \le 4.844819) = .90$$

Which can then be scaled back into kg, yielding $[4.346071, 22.961041]$, a reasonable interval for a 10 year forecast. As time continues to increase, our estimate will grow less accurate. However, this relatively narrow interval suggests that even farther in the future, our estimates, while less and less statistically significant, are still useful as a guide.

Task 2: Adjustable Model

Critical Considerations

We manipulated our first model to account for possible fluctuations with regard to …

- **Political Dynamics**
  - Stable or unstable relations with foreign countries could influence the effectiveness of implementing universal policies.
  - Similarly, the number of environmentalists involved within the bureaucratic systems could affect implementation.
  - Budget cuts or spending decided by a country's leader or government might impact our models.
- **Environmental Disruption**
  - Natural disasters may disturb conditions in the ozone layer, making it more susceptible to damage inflicted by ODS.[2]
    - Volcanic eruptions release especially harmful particles (sulfur) into the atmosphere.[6]
  - Wind patterns affect atmospheric temperatures.[2]
    - Extremely low temperatures (as in the arctic polar regions) allow ozone to break up easily.[27]
- **Demographic Trends**
  - Humans' awareness of the important role that ozone plays could spark more environmental activism.
    - Extreme amounts of UV exposure can be catastrophic to living organisms and their habitats by…

      … triggering an increase in skin cancer rates.

      … causing forms of human blindness.

      … affecting marine life food chain dynamics.

A Model with Restrictions

In light of global concern for the preservation of the ozone and life on Earth, we propose to establish an international body to implement ozone awareness education and enforce ozone conservation policies.

- This organization will be comprised of a coalition of countries formed to regulate CFC/Halon emissions around the world.
- The body would consist of environmentally conscious individuals who would be dispersed among the various governmental levels in each country (advisors to country leaders, cabinet members or adjudicators, etc.).
- These individuals would provide their assigned countries with experience in environmental politics. They would basically serve as ambassadors of ozone protection in their respective fields, supporting previously mentioned policies.
- The overall effect of this new body would be to actively promote the replacement of all CFC/Halon emissions with the more environmentally safe HCFC versions, and to do so by 2001 (see models).

Table 3 -- Forecasted Ozone Levels

year	Ozone (millions of kg)	year	ODS per year (million kg)
2000	3,000,000	**2011**	**2,999,600.25**
2001	**2,999,920.66**	**2012**	**2,999,590.60**
2002	**2,999,855.15**	**2013**	**2,999,582.64**
2003	**2,999,801.06**	**2014**	**2,999,576.07**
2004	**2,999,756.40**	**2015**	**2,999,570.64**
2005	**2,999,719.53**	**2016**	**2,999,566.16**
2006	**2,999,689.09**	**2017**	**2,999,562.46**
2007	**2,999,663.95**	**2018**	**2,999,559.41**
2008	**2,999,643.20**	**2019**	**2,999,556.89**
2009	**2,999,626.07**	**2020**	**2,999,554.81**
2010	**2,999,611.93**	*bold indicates forecasted*	

Due to the fact that HCFC's only cause 8.17% as much damage to the ozone as CFC/Halons, the rate of ozone depletion per unit of ODS is lowered to 1.04. The net effect on the ozone levels in the stratosphere is an overall decrease in ozone levels of 445.2 million kilograms of O_3 (see Table 3 and Appendix T3). This is compared with our projection of 4.48 billion kilograms of O_3 with no restrictions in place.

Overall Effect on Cancer Rates

One of the major effects of ozone depletion is a sizeable increase in skin cancer rates in humans. Our knowledge of a relationship between ultra-violet rays and these diseases ranges from an observed causal relationship with non-melanoma cancers to a loose correlation with melanoma skin cancer rates. However, there is a large body of work dedicated to demonstrating this very relationship.[29] The process of creating and destroying ozone is the natural mechanism in our stratosphere that protects humans from the Sun's harmful ultra-violet rays. As there is less O_3, there is an increase in the amount of ultra-violet radiation that makes it to the Earth's surface.

The average skin cancer rate among humans is 5 cases per million per year as of 2000 with an estimated growth rate of 7.3% per year with no restrictions, and 5.5% per year with the emission restrictions.[10] Combining this with our knowledge of the amount of O_3 present in the stratosphere at the same time, we can extrapolate a connection between the amount of ozone present and skin cancer rates.

Using our forecasting model, we can use this growth rate to estimate the difference between the two models in the future amounts of skin cancer, melanoma and non-melanoma cancers, and the amount of deaths.

If we let $m_0 = n_0 = 30000$ (where n_0 and m_0 represent the deaths per year due to skin cancer with and without restrictions to ODS emissions, respectively), then:

$$m_t = m_0(1.073)^t \Rightarrow m_{20} = 30000(1.073)^{20} = 122776.6259$$
$$n_t = n_0(1.055)^t \Rightarrow n_{20} = 30000(1.055)^{20} = 87532.72472$$

The overall incidence of cancer can be broken up into two different groups: non-melanoma and melanoma. While non-melanoma cancer occurs much more frequently than melanoma cancer, non-melanoma cancer tends to be relatively more benign.

Table 4 -- Non-Melanoma and Melanoma Cancer Ratios

Cancer Types	Relative Incidence Rates	Death Rates Once Acquired	Total Fatalities per Cancer
Non-Melanoma	99.5%	1%	.00995
Melanoma	.5%	24%	.00120
		Deaths per Cancer	.01115

Therefore, 1.115% of all skin cancer cases are fatal (see Table 4). This translates to:

$$(122776.6259)(.01115) = 1368.959379 \text{ deaths per year}$$
$$(87532.72472)(.01115) = 975.9898806 \text{ deaths per year}$$

Thus, the total difference in deaths per year resulting from skin cancer in an emissions restricted society compared with a non-restrictive society in the year 2020 is 392.9695 deaths.

Overall Effect on the Economy

Although the reduction in ozone depletion due to the replacement of CFC/Halon emissions with HCFCs leads to a decline in cancer related deaths per year, the effects are not entirely beneficial. In industry economics, the price of any good (and thus the overall demand for that good) is strongly related to the cost of inputs. In this respect, replacing the more nefarious ODS chemicals with the safer HCFC's can have negative implications as well. One kilogram of HCFC costs on average 2.75 times as much as a kilogram of CFC/Halon.[12] If we multiply each molecule by its forecasted usage, we can get an idea for the total difference in overall costs of using CFC/Halon or HCFC's.

Table 5 -- Cost Per Kilogram of CFC/Halon and HCFC molecules

	Cost ($/kg)	Total Forecasted Usage	Total Forecasted Cost
CFC/Halon	2	153012.009	306024.0180
HCFC	5.5	153012.009	841566.0495
		Total Difference in Costs	535542.0315

This is equivalent to a 63.64% increase in cost between the two differing plans (see Table 5). This great of a discrepancy would have far reaching and profound implications in the world macro economy as a whole. The market shock produced by this decrease in the supply schedule would cause a quantity decrease in demand for products that use or emit CFC/Halon molecules. This would constrict the budget constraint of the average consumer in the economy and force effective purchasing power of currency down. However, in the long run, the economy would return to equilibrium at a new lower quantity and a higher price.

Policies: Tasks 3 Through Task 5

Task 3: Management Policy

In developing policies that could prevent further ozone damage, we considered several areas of interest.

1. **Law Enforcement**

   a. The leader of the government should support and enforce laws that protect the ozone layer

   b. A smaller branch or department within the government could help the leader make decisions on environmental issues.

   Assumption:
   - Leader or government is invested in about promoting environment-friendly policies.

2. **Education**

   a. Individuals should become aware of the adverse effects of UV ray exposure and the use of ozone depleting substances.

   Assumptions:
   - News and up-to-date information is readily available to the general public.
   - The general population cares about the issue enough to notice the posted news and information.

3. **ODS Regulations**

   a. Producers and factories should be limited to the amount of pollutants they can release.

      - A filtering system could be established to reduce the amount of pollutants being released.

   b. Fines could be administered for those that exceed the limits.

   Assumptions:
   - Companies are willing to comply to the restrictions.
   - Production amounts would not be affected.

4. **Transportation**

   a. A carpooling agency or program should be designed to decrease exhaust emissions from cars.

   b. Public transportation passes could be given out by companies to their employees.

   c. Public transportation should be more accessible (more subways, buses, trains, etc).

   Assumptions:
   - People are willing to divulge personal information to the carpooling agency.
   - Companies have the financial capability to provide transportation passes.

Task 4: Security Policy

The depletion of the ozone layer is a worldwide concern. We felt that the model in Task 2 clearly represented what could occur with restrictions of CFC use. The net loss in ozone levels (see Table 3) is 445.1945 million kilograms of O_3, as opposed to 4.4798 billion kilograms of O_3 without restrictions. With an international environmental coalition, we can execute policies that may thwart further destruction of the ozone layer.

Task 5: Environmental Policy

We recognize several environmental dynamics influencing the effectiveness of our previously discussed policies:

- Volcanic eruptions
  - Particles released into the air when a volcano erupts accelerate ozone depletion.[24]
- Weather patterns
  - Lower temperatures cause increases in ozone depletion.
    - Winds cycles affect temperatures in the atmosphere
  - Lower altitudes register lower Dobson readings.[2]

However, taking these factors into account, we concluded that developing a policy regulating natural or uncontrollable processes would be impossible to execute in the time allotted.

Task 6: Ozone Layer Alternatives

Resolution of the ozone problem is not so easily entreated. Since ozone is very toxic to humans, manmade production of O_3 could be very dangerous. The preventative methods of preserving ozone (by restricting ODS use) would probably be the best and safest approach. Even though it may require a long period of time, the natural process of creating O_3 is most likely the only alternative we have at this moment.

Some preventative measures are:

- ODS and Halon replacement/using products that are "ozone friendly."
- Public awareness or education.
- Government and international regulation towards ozone protection.

The restricted model shows that policy making and prevention can reduce the effects of ODS use. With restrictions, there is a loss of 445.19 million kilograms of O_3, and without restrictions, we lose a significant 4.48 billion kilograms. As the results indicate, policies and preventative measures can be very beneficial to the preservation of the ozone layer.

We know that the high risks of developing skin cancer can be connected to the deterioration of the ozone layer. Since certain lifestyles involve long periods of activity in sunlight, precautions should be taken to avoid overexposure to UV rays. Protective sun wear (such as sun block and sunglasses) can provide a temporary shield from ultra-violet radiation and decrease the chances of developing skin cancer.

Depending on technological resources and advances, we may someday develop a mechanism capable of replacing ozone. Maybe further research could be performed to generate O_3 by a non-toxic and non-pollutant system. Or perhaps some day an unknown element or gas that increases the regeneration rates of O_3 may be discovered, and used to help rebuild the ozone layer.

We feel that these concepts have potential, and that further exploration is not only valuable, but necessary. As an indispensable resource, appropriate measures must be taken to ensure its protection and security. We feel that our suggestions, while rudimentary, elicit important facets of research conducive to future ozone repair.

Note:
In compiling this project we obtained information from a vast array of sources. Some provided statistical figures and numerical data; some supplied deeper understanding into the various dynamics of our resource; and some were referenced for the sole purpose of obtaining certain knowledge of a discipline related to our comprehensive project.
We wished to incorporate the numerous sources we felt supplemented our credibility in the bibliography, although there are no references to these in the text (since we did not directly cite them). Works from which we extracted facts or data, however, are denoted by superscript numbers corresponding to the alphabetical arrangement of our resources. The following is a comprehensive index of every work we referenced in the execution of this project.

Reference Materials

1. Abarca, Jaime F., and Casiccia, Claudio C. "Skin cancer and ultraviolet-B radiation under the Antarctic oxone hole: southern Chile, 1987 – 2000." *Photodermatology, Photoimmunology, & Photomedicine.* (Blackwell Munksguard, 2002).

2. Abott, Patrick L. *Natural Disasters*. (San Diego: McGraw-Hill, 2004). 4th ed.

3. About.com: Geography. "World Population Growth." (2004). Retrieved February 4th, 2005, from http://geography.about.com/od/obtainpopulationdata/a/worldpopulation.html

4. Bitton, Gabriel. *Formula Handbook for Environmental Engineers and Scientists.* (New York: John Wiley & Sons, Inc., 1998).

5. Bodeker, G.E. "A New Zealand Perspective on Global Ozone." National Institute of Water and Atmospheric Research (NIWA). (2001). Retrieved February 4th, 2005, from http://www.niwa.co.nz/rc/atmos/uvconference/Bodeker1.pdf.

6. Busman, Joelle S., and Belen, Cary. "The Ozone Layer: Important Components of Ozone Education." (no date). Retrieved February 4th, 2005, from http://www.umich.edu/~gs265/society/ozone.htm.

7. Dobson Report. "Ozone Measurement and Distribution." NOAA Climate Monitoring and Diagnostics Laboratory. (1995). Retrieved February 4th, 2005, from http://www.cmdl.noaa.gov/ozwv/dobson/papers/wmobro/ozone.html.

8. e-Digest of Environmental Statistics. "Table 11 Production and consumption of ozone depleting substances in the UK and globally: 1986-2002." Department for Environment, Food and Rural Affairs, and UNEP. (2004). Retrieved February 4th, 2005, from http://www.defra.gov.uk/environment/statistics/index.htm.

9. Envis Node. "Acid Rain and Atmospheric Pollutant Modeling." Ministry of Environment and Forest. (1999). Retrieved February 4th, 2005, from http://envis.tropmet.res.in/.

10. Fears, Thomas R., Scotto, Joseph, and Schneiderman, Marvin A. "Skin Cancer, Melanoma, and Sunlight." *American Journal of Public Health.* (May, 1976). Vol. 66 Issue 5. Retrieved February 5th, 2005, from http://search.epnet.com/login.aspx?direct=true&db=aph&an=5666391.

11. Goodchild, Michael F., Parks, Bradley O., and Stayaert, Louis T. *Environmental Modeling with GIS.* (New York: Oxford University Press, 1993).

12. Greenpeace Methodology. "The Ozone Layer Destroyers: Whose Chlorine and Bromine Is

It?" (2004) Retrieved February 4th, 2005, from http://archive.greenpeace.org/ozone/chlorine/3chlor.html.

13. Hill, R. Carter, Griffiths, William E., and Judge, George G. *Undergraduate Econometrics.* (New York: John Wiley & Sons, Inc., 1997).

14. Manahan, Stanley E. *Environmental Chemistry.* (Boca Raton: Lewis Publishers, 2000). 7th ed.

15. Olmsted, John III, and Williams, Gregory M. *Chemistry.* (New York: John Wiley & Sons, Inc., 2002). 3rd ed.

16. Ozone Depletion. "Class I Ozone-Depleting Substances," and "Class II Ozone-Depleting Substances." U.S. EPA. (2005). Retrieved February 4th, 2005, from http://www.epa.gov/ozone/ods.html.

17. Ozone Depletion Glossary. United States Environmental Protection Agency (U.S. EPA), 2004. Obtained February 4th, 2005, from http://www.epa.gov/ozone/defns.html.

18. "Ozone Depletion [South Pole Ozone]." Climate Monitoring and Diagnostics Lab (CMDL), NOAA. (no date). Retrieved February 5th, 2005, from http://www.cmdl.noaa.gov/ozone.html.

19. Pierce, J. Jeffrey, Weiner, Ruth F., and Vesilind, P. Aarne. *Environmental Pollution and Control.* (Boston: Butterworth-Heinemann, 1998). 4th ed.

20. "Production and Consumption of Ozone Depleting Substances under the Montreal Protocol, 1986 – 2002." UNEP. (2002). Retrieved February 4th, 2005, from http://www.unep.org/ozone.

21. Waite, Thomas D., and Freeman, Neil J. *Mathematics of Environmental Processes.* (Lexington: Lexington Books, 1977).

22. Ramsey, Fred L., and Schafer, Daniel W. *The Statistical Sleuth: A Course in Methods of Data Analysis.* (Pacific Grove: Duxbury, 2002). 2nd ed.

23. Ray, Bill T. *Environmental Engineering.* (Carbondale: PWS Publishing, 1995).

24. Scientific Assessment of Ozone Depletion: 2002. "Twenty Questions and Answers About the Ozone Layer." National Oceanic and Atmospheric Administration (NOAA), National Aeronautics and Space Administration (NASA), United Nations Environment Programme (UNEP), World Meteorology Organization (WMO), and European Commission, 2002. Obtained February 4th, 2005, from http://www.al.noaa.gov/WWWHD/pubdocs/assessment02/Q&As.html.

25. Seniz, H. "Service Module 7: Montreal Protocol – Overview." United Nations Industrial

Development Organization (UNIDO). (no date). Retrieved February 4th, 2005, from http://www.unido.org/doc/5072.

26. Stewart, James. *Calculus Concepts and Contexts*. (Pacific Grove: Brooks/Cole, 2001). 2nd ed.

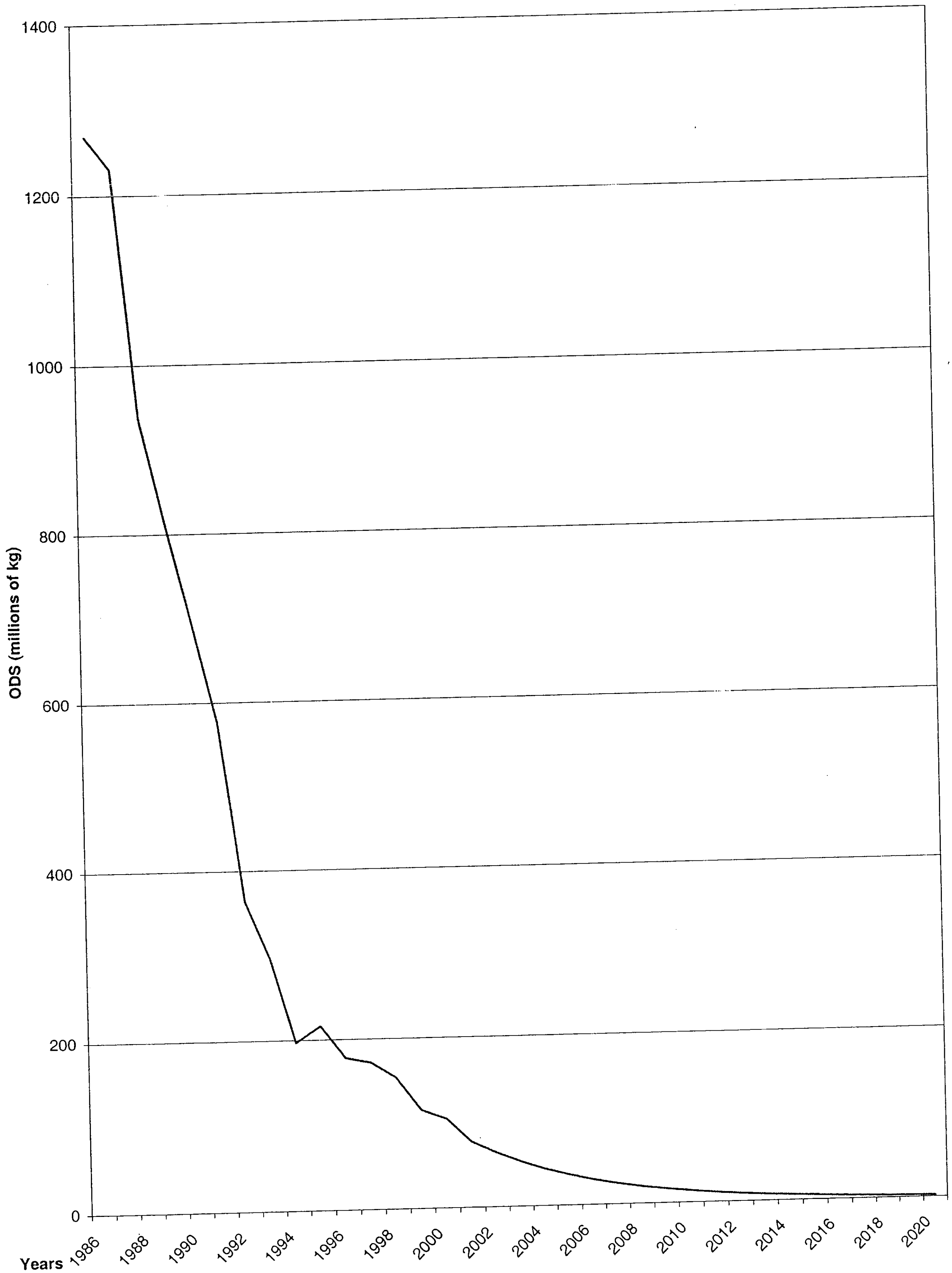
27. The Ozone Hole Tour. "Part III. The Science of the Ozone Hole." University of Cambridge Centre for Atmospheric Science. (no date). Retrieved February 4th, 2005, from http://www.atm.ch.cam.ac.uk/tour/part3.html.

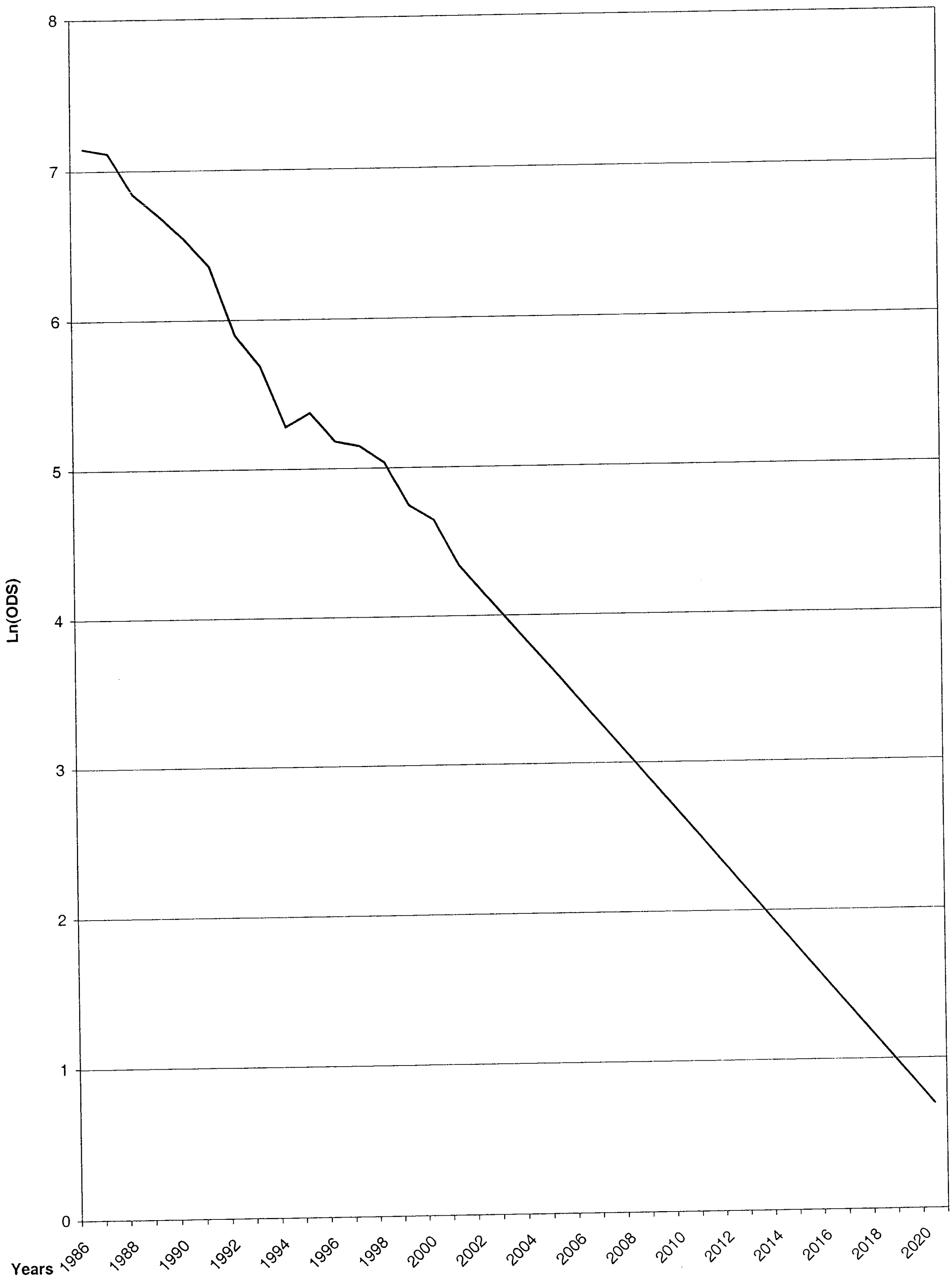
28. Total Ozone Mapping Spectrometer. "Ozone Levels Measured by the TOMS." (no date) Retrieved February 6th, 2005, from http://www.earthobservatory.nasa.gov.

29. Velders, G.J.M., Slaper, H., Pearce, D.W., and Howarth, A. "Technical Report on Stratospheric Ozone Depletion." (no date). Retrieved February 5th, 2005, from http://europa.eu.int/comm/environment/enveco/priority_study/stratospheric_ozone.pdf.

30. Welch, Charles. "The Ozone Hole." The British Antarctic Study, NASA, Environment Canada, UNEP, and EPA, for The Ozone Hole, Inc., 2005. Retrieved February 4th, 2005, from http://www.theozonehole.com.

Appendix C.1 -- ODS Consumption





8
7
6
5
4
3
2
1
0
Ln(ODS)
Years
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018
2020

year	ODS rate (millions)	ln(ODS)	Ozone (megatons)
1986	1269	7.14598447	3092.78351
1987	1231	7.11558213	
1988	937	6.84268328	
1989	814	6.70196037	
1990	696	6.54534966	
1991	577	6.35784227	
1992	364	5.89715387	
1993	296	5.69035945	
1994	197	5.28320373	
1995	216	5.37527841	
1996	178	5.18178355	
1997	172	5.14749448	
1998	154	5.0369526	
1999	115	4.74493213	
2000	104	4.6443909	3000
2001	**76.5849054**	**4.3384**	**2999.02845**
2002	**63.2313359**	**4.1468**	**2998.22633**
2003	**52.2061341**	**3.9552**	**2997.56405**
2004	**43.1162519**	**3.7636**	**2997.01709**
2005	**35.5876974**	**3.572**	**2996.56564**
2006	**29.3825218**	**3.3804**	**2996.1929**
2007	**24.2592988**	**3.1888**	**2995.88515**
2008	**20.0293761**	**2.9972**	**2995.63106**
2009	**16.5369951**	**2.8056**	**2995.42128**
2010	**13.6535556**	**2.614**	**2995.24807**
2011	**11.2728818**	**2.4224**	**2995.10507**
2012	**9.30730895**	**2.2308**	**2994.987**
2013	**7.68445917**	**2.0392**	**2994.88952**
2014	**6.34457426**	**1.8476**	**2994.80903**
2015	**5.23831562**	**1.656**	**2994.74258**
2016	**4.32494749**	**1.4644**	**2994.68771**
2017	**3.57083692**	**1.2728**	**2994.64241**
2018	**2.94821529**	**1.0812**	**2994.60501**
2019	**2.43415579**	**0.8896**	**2994.57413**
2020	**2.00972923**	**0.698**	**2994.54864**
		Overall Change in Ozone	4.4798074

ln(y)bar	xbar	(x-xbar)	(x-xbar)^2	y-ybar	(x-xbar)(y-ybar)
5.847397	1993	-7	49	1.298588	-9.090114
		-6	36	1.268185	-7.609112
		-5	25	0.995287	-4.976433
		-4	16	0.854564	-3.418254
		-3	9	0.697953	-2.093859
		-2	4	0.510446	-1.020891
		-1	1	0.049757	-0.049757
		0	0	-0.157037	0
		1	1	-0.564193	-0.564193
		2	4	-0.472118	-0.944237
		3	9	-0.665613	-1.99684
		4	16	-0.699902	-2.799609
		5	25	-0.810444	-4.052221
		6	36	-1.102465	-6.614788
		7	49	-1.203006	-8.421041
		Sums	280		-53.65135
		Beta1Hat	-0.191612		
		Beta0Hat	387.73		

year	HCFC rate (millions)	Total ODS	Ozone (megatons)	
1986	1072		3092.78351	
1987	1046			
1988	764			
1989	664			
1990	591			
1991	506			
1992	338			
1993	254			
1994	152			
1995	159			
1996	147			
1997	147			
1998	133			
1999	101			
2000	94		3000000	
2001	**76.5849054**	**7390.58491**	**2999920.66**	
2002	**63.2313359**	**7435.81624**	**2999855.15**	
2003	**52.2061341**	**7506.02238**	**2999801.06**	
2004	**43.1162519**	**7549.13863**	**2999756.4**	
2005	**35.5876974**	**7584.72633**	**2999719.53**	
2006	**29.3825218**	**7614.10885**	**2999689.09**	
2007	**24.2592988**	**7638.36815**	**2999663.95**	
2008	**20.0293761**	**7658.39752**	**2999643.2**	
2009	**16.5369951**	**7674.93452**	**2999626.07**	
2010	**13.6535556**	**7688.58807**	**2999611.93**	
2011	**11.2728818**	**7699.86095**	**2999600.25**	
2012	**9.30730895**	**7709.16826**	**2999590.6**	
2013	**7.68445917**	**7716.85272**	**2999582.64**	
2014	**6.34457426**	**7723.1973**	**2999576.07**	
2015	**5.23831562**	**7728.43561**	**2999570.64**	
2016	**4.32494749**	**7732.76056**	**2999566.16**	
2017	**3.57083692**	**7736.3314**	**2999562.46**	
2018	**2.94821529**	**7739.27961**	**2999559.41**	
2019	**2.43415579**	**7741.71377**	**2999556.89**	
2020	**2.00972923**	**7743.7235**	**2999554.81**	
	Total Amount of CFC	**153012.009**	**445.194457**	Overall Change in Ozone