Analysis and Optimization of Hand Moved Irrigation Systems

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Contents

1 Introduction

As a team of analysts working in cooperation with certain agriculturalists, we were tasked with finding an algorithm designed to minimize the time and labor requirements inherent with a “hand move” irrigation system. Our goal was to find the optimal number of sprinkler heads and the placement of those sprinklers to efficiently, and as uniformly as possible, irrigate a field. This concept is general and expandable, so as a test scenario, we worked to optimize the irrigation of a plot of land 80 meters by 30 meters in dimension.

We also were asked to try to minimize the initial investment on the part of the agriculturalist so we needed to keep the structure as simple as possible. To this end, we based our models on a pipe system 20 meters in length, with an inner diameter of 10 centimeters. The sprinklers have an inner diameter of 0.6 cm. This pipe is assumed to be constructed out of a lightweight and inexpensive material, such as aluminum. We also assumed that the entire system was this one piece and there are no lateral sections.

To address the problem of optimal sprinkler placement and irrigation scheduling for this particular hand-move sprinkler system, on this particular field, we explored several modeling concepts. These concepts are outlined in the pages ahead.

2 Some Basic Assumptions

In all that follows, we had some assumptions that remained more or less consistent.

2.1 Environmental Considerations

- We assume that natural precipitation is negligible for the purposes of our proposed watering schedules.
- Wind conditions are low, and temperatures are moderate so as to minimize the effects of evaporation.
- To account for evaporation we found that sprinklers are usually between 60% and 80% efficient[15], so we used an efficiency rating of 70% for our models.
- The field is assumed to be level, with no contours or irregularities.
- Water temperature is assumed to be an average of 20° C. Air resistance is negligible in terms of the projectile motion of individual water droplets. We assume normal projectile motion given by

$$\begin{aligned} x &= x_0 + v_0 t\cos(\theta), \\ y &= y_0 + v_0 t\sin(\theta) - \frac{1}{2}gt^2. \end{aligned}$$

- Velocity is related to flow rate by the equation

$$v = \frac{4 * (\text{flow rate})}{\pi D^2}$$

  where D represents diameter. This formula was used as v_0 in the equations above.

- The crops being irrigated have an irrigation requirement of 2.00 cm every 4 days. However, the soil has an absorption limit such that the application rate of the water cannot exceed 0.75 cm per hour, meaning that any application rate exceeding this will result in runoff.

2.2 Irrigation System

- The system is configured such that the sprinkler heads are 1 meter above whatever crop is being irrigated. This generalizes to assuming that the sprinkler is 1 meter above the ground. Watering regions and wetting radii calculations are based on this. Our research indicated that "hand move" systems are operable for most crops, but most suited to low lying crops[22].
- The water pressure remained uniform throughout the system, and the system is fully capable of withstanding the pressure. The pressure was given to us as 420 kilopascals (KPa).
- The flow rate remains constant from the source (pump) throughout the pipes. This was given to us as 150 liters per minute.
- The pipe system is closed, meaning that the only way for water to escape is through the sprinklers.
- The sprinkler heads our models are based upon are rotating impact type sprinklers, and are presumed to be adjustable. This means the operator can in some measure control the wetting radius, within certain limits.
- The operator can control the angle of rotation.
- The sprinkler heads are constructed such that the water streams out at a 30° angle of inclination.

2.3 Other Concerns

Finally, we assume there are no crop circles or other types of unexplainable phenomena, especially those caused by extraterrestrial activity, since this might affect the relative efficiency of our models. We feel this is safe due to the extremely small probability of interstellar travel.

3 The Hasty Model

To begin, in addition to the assumptions outlined in the previous section, we made an additional assumption that the application of water to the area of influence for each sprinkler head is uniform. This means that the average application rate is true for all the area in the wetting region.

3.1 The Structure of the System

Our task is to irrigate a field that is 80 meters by 30 meters in dimension. For our initial model, we constructed the irrigation system so that the main pipe is oriented parallel to the short edge. There are a total of two sprinkler heads, each placed 5 meters from the end of the main pipe. The wetting radius for each sprinkler is set at 10 meters, and the angle of rotation is 180°. The area of influence for this configuration is depicted below (Figure 1).

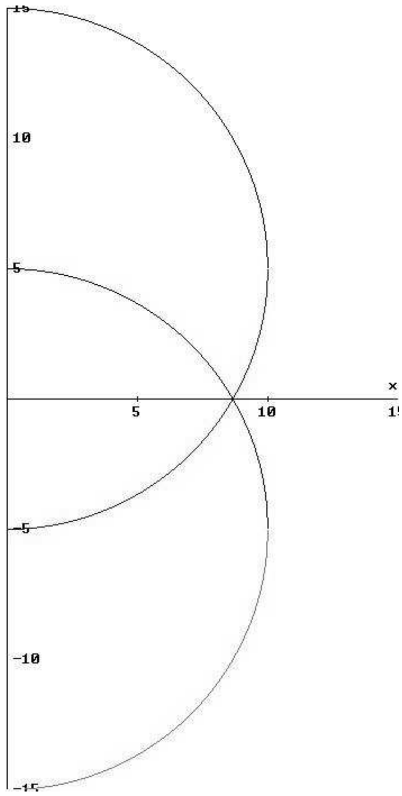

Figure 1

3.2 Analysis of the System

To analyze the efficacy of this configuration, we had to determine the application rate per sprinkler to the wetted region. This was calculated by measuring the amount of water passing through the sprinkler, or the precipitation rate Pr, given by

$$Pr = \frac{(96.3)(\text{nozzle flow rate})}{(\text{area covered})}, \tag{1}$$

where nozzle flow rate is measured in gallons per minute (gpm), and area is in square feet[15]. The determination of nozzle flow rate is based on nozzle size and the pressure of the system. Hill[15] offers a data table mapping nozzle size in inches, and pressure in pounds per square inch (psi) to nozzle flow rates in gallons per minute. Converting units, we found that 0.6 cm is approximately 7/32 in. and 420 KPa is approximately 60 psi. Using this, we found that our nozzle flow rate is about 10.5 gpm. The area covered by one sprinkler head is given by

$$A = \frac{\pi r^2}{2}. \tag{2}$$

Here, $r = 32.81$ ft., so the area covered is 1690.96 sq. ft. Combining equations (1) and (2), we have

$$Pr = \frac{(96.3)(10.5)}{(1690.96)} = 0.598 \quad \text{in. per hour.}$$

Converting this back to metric gives 1.52 cm/hr. Now to find the application rate we take the precipitation rate and multiply by the application efficiency. Using an efficiency of 0.70, we get

$$Ar = (\text{efficiency})(Pr) = 1.06 \quad \text{cm per hour.} \tag{3}$$

Remember, this is for each sprinkler, so in areas where the wetting regions overlap, we must add the application rates together. So the area of overlap is receiving an application rate of 2.12 cm per hour.

3.3 Scheduling

In order to not exceed 0.75 cm per hour in the overlap region, the operator would be forced to move the sprinkler system every 21 minutes. When the system is moved, the region receiving the largest amount of water must not be watered again. So the minimum distance to move the sprinkler system is where the two respective wetting areas intersect. The point of intersection is 8.66 meters from the system, so the operator would be forced to move the system 8.66 meters every 21 minutes.

3.4 Concluding Discussion

The positive aspect of this system and schedule is that the initial investment is minimized, since the irrigation system is based only on the main pipe and two sprinkler heads.

However, the negative aspects far outweigh the positive. First, the application rate is too great, so water is being wasted due to runoff. Second, we are not actually watering the entire field. Moving the sprinkler system 8.66 meters means that there are uncovered areas near the edges of the field. Also, the orientation of the main pipe in relationship to the field implies that the system must be disassembled in order to move it. If the operator were to attempt to simply drag the pipe to its new location, part of the field would be damaged or destroyed in the process. If the system consisted of the main pipe and several lateral pipes, then disassembly and reassembly would be unavoidable. In a simple system like ours, this is an unacceptable labor requirement.

In addition, due to the short duration of watering times, the operator would be forced to stand by and observe. When the required time has elapsed, the operator would then need to move the system, which could take quite some time. Conceivably, the operator could spend an entire day, perhaps two, doing nothing but watching the field be irrigated. This is an unacceptable time requirement.

Clearly, based on the extreme time and labor considerations, along with the fact that some parts of the field are over-irrigated, and other parts remain dry, this is not in the best interests of the agriculturalist.

4 Strike Two

The model described in the previous section had certain drawbacks. Some parts of the field were covered too well, and other parts were not covered at all. Also, the time and labor requirements were too extreme to be feasible. In an attempt to correct this, we developed the following configuration for the irrigation system. We divided the field into four watering sections, the dimensions of each being 20 meters by 30 meters.

4.1 The Structure of the System

There are four sprinklers in the system: one at each end of the pipe set, and two at approximately the middle of the pipe. The main pipe is oriented such that it is perpendicular to the short edge of the field, with sprinkler heads at each end of the pipe. Their watering radius is set at 15 meters with a rotational angle of 180°. Another two heads are situated in the center of the pipe; their placement is such that they are equidistant from the actual center, but far enough apart from one another that the rotational operation of one sprinkler does not interfere with the other. The central sprinkler heads are set so that their radius is 18 meters and their rotational angle is 60°. The wetting region for each sprinkler head is depicted below.

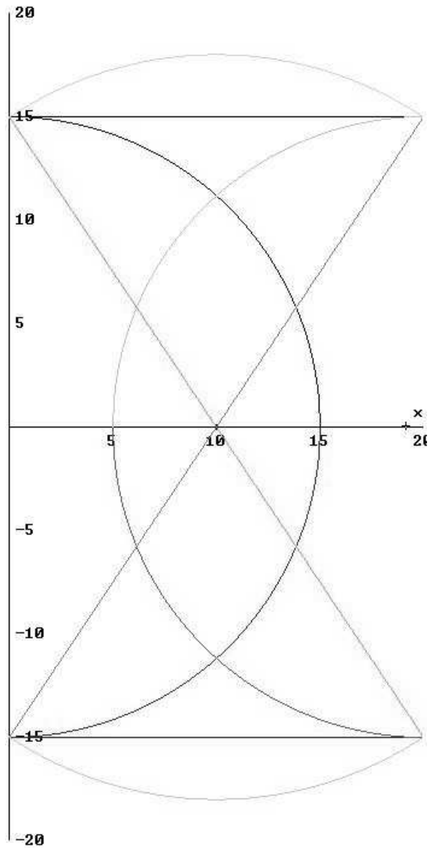

Figure 2.

4.2 Motivation

The basis for this sprinkler configuration was simply to achieve total coverage of the field. Initially, we were not overly concerned with application rates when creating this model, we simply wanted to ensure that no part of the field was outside of the wetting region of at least one of the sprinkler heads. At this point, we were also concerned by the thought that any of the water we were using to irrigate might actually fall outside of the established boundaries.

4.3 Analysis of the System

As before, we first determined the rate of flow for the individual sprinkler heads to provide information on the application rate of the new wetted region. We first needed to calculate the areas covered by both the 180° and 60° rotating sprinkler heads, given by

$$A_{180^\circ} = \frac{\pi 49.22^2}{2} = 3804.65,$$

and

$$A_{60^\circ} = \frac{\pi 59.1^2}{6} = 1826.17,$$

with our radii measured in feet. We could then calculate our new respective precipitation rates,

$$Pr_{180^\circ} = \frac{(96.3)(10.5)}{(3804.65)} = 0.2658 \quad \text{in. per hour},$$

and

$$Pr_{60^\circ} = \frac{(96.3)(10.5)}{(1826.17)} = 0.5537 \quad \text{in. per hour}.$$

Converting these values to centimeters gives us

$$Pr_{180^\circ} = .6751 \quad \text{cm per hour},$$

and

$$Pr_{60^\circ} = 1.406 \quad \text{cm per hour}.$$

To find the associated application rates we use equation [3] and multiply our precipitation rate by the application efficiency of 70%:

$$Ar_{180^\circ} = (.70)(.6751) = .4726 \text{ cm per hour,}$$

and

$$Ar_{60^\circ} = (.70)(1.406) = .9845 \text{ cm per hour.}$$

Thus in the areas of maximum overlap, our application rate is

$$2Ar_{180^\circ} + Ar_{60^\circ} = 2 * .4726 + .9845 = 1.930 \text{ cm per hour,}$$

so we are obviously exceeding our bound of 0.75 cm per hour here. In our regions of non-overlap, we fail to satisfy our requirement of 2.00 cm every four days:

$$Ar_{180^\circ} = .4726 \text{ cm} * 4 \text{ days} = 1.8904 < 2.00 \text{ cm every four days,}$$

$$Ar_{60^\circ} = .9845 \text{ cm} * 4 \text{ days} = 3.938 > 2.00 \text{ cm every four days.}$$

Yet if we shortened our watering time so that the areas of overlap receive $\leq$ 0.75 cm per hour, the regions with single sprinkler coverage would receive even *less* water and never achieve required irrigation levels.

For what it was worth, we determined the efficiency of this model by calculating what percentage of the wetting area actually lay within the field dimensions. We found the approximate area A_{60° of the field irrigated by each of the sprinklers that oscillate 60°, and subtracted this from the wetting area of these spray nozzles W_{60° to approximate the area *outside* of the field that is being watered, O_{60°:

$$A_{60^\circ} = 1/2 \text{ base} * \text{ height} = 1/2 * 20 * 15 = 150 \text{ sq. m},$$

$$W_{60^\circ} = 1/6 * \pi r^2 = 1/6 * \pi 18^2 = 169.65 \text{ sq. m},$$

and

$$O_{60^\circ} = W_{60^\circ} - A_{60^\circ} = 169.65 - 150 = 19.65 \text{ sq. m}.$$

Using this watering scheme to irrigate each quarter of the field, the wetting area that lay outside the field is given by

$$2 * O_{60^\circ} = 2 * 19.65 = 39.29 \text{ sq. m},$$

and the total area outside the field that is watered is

$$TO_{60^\circ} = 4 * 2 * O_{60^\circ} = 314.34 \text{ sq. m}.$$

Since the entire area of the field is

$$T_A = 30 * 80 = 2400 \text{ sq. m},$$

the efficiency is simply

$$\frac{T_A - TO_{60^\circ}}{T_A} = \frac{2400 - 314.34}{2400} = 87\%.$$

4.4 Scheduling

The discrepancies inherent in our second model ultimately prevented us from developing a satisfactory algorithm for watering. The operator would only be able to water each section for approximately nine minutes, before having to relocate to the next section. He would then have to repeat the process every nine minutes; watering the field in this way would require constant attention in a manner similar to our first scenario.

4.5 Concluding Discussion

We embarked on this model having developed some fundamentally hindering assumptions based on the difficulties encountered in our hasty model. Our second watering scheme indeed dispersed water to all areas in each of the four sections, but we sacrificed the gross over-watering of some areas in order to irrigate the entire field. In an attempt to justify our assumptions, we explored the idea of accounting for the density variance of each sprinkler's water distribution; but this only further illustrated the inadequacy of the model.

We did, however, develop some particularly useful conventions in assembling our second model, such as dividing the periphery into four vertical sections. Moreover, the perpendicular arrangement of the pipe to the 30 meter-long field edge proved much more efficient than our first model, as this allowed ready movement of the assembled pipe to each quadrant with minimal time and energy expenditure.

5 Third Time's the Charm

The previous two scenarios both had certain drawbacks. These were based mostly on the application rate of water in certain areas of the field and the labor requirements for moving the system, although the second configuration vastly improved upon the first in this regard. However, the second arrangement still had the issue of excessively watering some parts of the field, and the subsequent under-watering of other parts of the field. This meant that we had to reconfigure the system in such a way so as to address all these issues.

One of the main issues to address was the assumption of uniform distribution of water to all the area of the wetting region. This actually turned out to be a giant leap, because in practice the distribution is not uniform. The purpose of a sprinkler is to scatter a stream of water over an area. During the scattering process, the stream of water is broken up into many droplets of varying size, usually ranging from 0.5 mm to 4 mm in diameter[26]. The smaller drops do not travel as far as the bigger drops, and a proper sprinkler is going to ensure that bigger drops are relatively rare. So more water falls near the sprinkler, as depicted below (Figure 3).

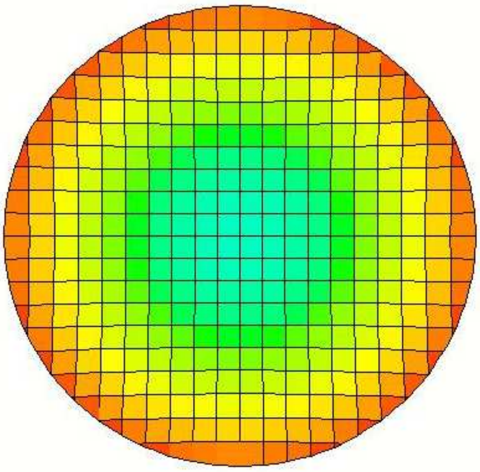

Figure 3

We made the assumption that the distribution of water was approximately paraboloidal[26]. As before, we divided the field into four 20 meter by 30 meter sections.

5.1 The Structure of the System

The system consists of a single main pipe 20 meters in length. The orientation of the main pipe is perpendicular to the short edge, and equidistant from each of the long edges of the field. The sprinkler heads are to be positioned at each end. The watering radius is set at 20 meters, and the rotational angle is 180°. The wetting regions are as depicted below (Figure 4).

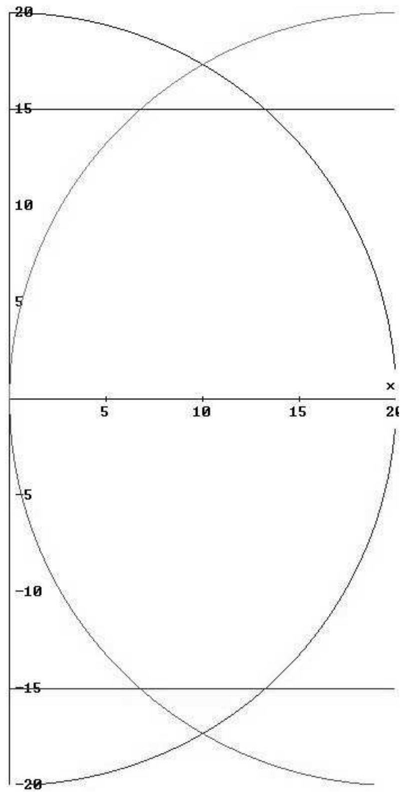

Figure 4

5.2 Analysis of the System

A thorough analysis of this configuration required a great deal of computation. We needed to determine the application rates based on a paraboloidal distribution, since maximum depths vary with distance from the sprinklers. We were concerned that the outer edges of the wetting regions occasionally fell outside of the field. We needed to figure out how much water was falling outside of the field, because water conservation should be a factor. We assumed that the distribution of the water, based on the factors outlined above, would resemble the figure below (Figure 5). The curvature has been exaggerated to show detail.

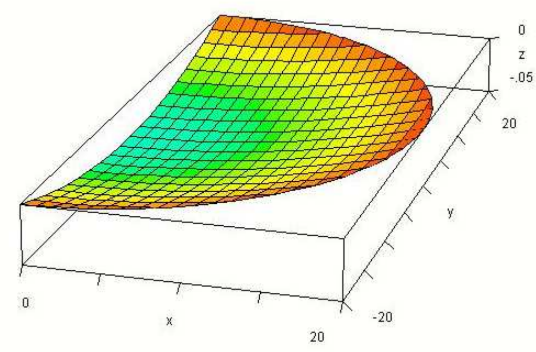

Figure 5

Since we assumed a paraboloidal distribution of water we have a parabolic cross-section. Using this we were able to calculate a maximum depth, from which we could determine the equation of the surface. We began by finding the application rate of the sprinklers in this configuration, which is

$$Ar = 0.2658 \quad \text{cm per hour.}$$

Employing some basic integral calculus, we found that on the function

$$x = \sqrt{y},$$

the line at which the area above is equal to the area below, or

$$\int_0^a \sqrt{y}dy = \int_a^1 \sqrt{y}dy,$$

was approximately at $y \approx .63$, meaning that the average value is approximately 63% of the maximum value. With these calculations we determined the maximum depth, based on one hour of watering, to be

$$d_{max} = 0.422 \quad \text{cm}.$$

This is an extreme point, so we used this as our z-intercept, assuming the cross-section of the surface to be where it intersects the yz-plane. The outer points of this cross-section correspond to the outer radius of the wetting region. Given these three points of a parabola, we then used the method of Lagrange interpolating polynomials to derive the equation of the parabolic cross-section, and then generalized this to obtain the equation of the surface.

We let our interpolation points be $(x_o, f(x_0)) = (-20, 0)$; $(x_1, f(x_1)) = (0, -.00422)$; and $(x_2, f(x_2)) = (20, 0)$. Given that our interpolating polynomial has form

$$P(x) = L_o(x)f(x_o) + L_1(x)f(x_1) + L_2(x)f(x_2), \tag{4}$$

with

$$L_o(x) = \frac{(x-x_1)(x-x_2)}{(x_o-x_1)(x_o-x_2)} \text{ , and} f(x_o) = 0,$$

$$L_1(x) = \frac{(x-x_o)(x-x_2)}{(x_1-x_o)(x_1-x_2)} \text{ , and} f(x_1) = -.00422,$$

$$L_2(x) = \frac{(x-x_o)(x-x_1)}{(x_2-x_o)(x_2-x_1)} \text{ , and} f(x_2) = 0.$$

From [4] we obtain

$$P(x) = 0 + \frac{(x-(-20))(x-20)}{(0-(-20))(0-20)} * (-.00422) = \frac{x^2-400}{400} * (-.00422) + 0,$$

(since the coefficients of $L_o(x)f(x_o)$ and $L_2(x)f(x_2)$ are zero) and finally

$$P(x) = 1.055 \times 10^{-5}x^2 - .00422. \tag{5}$$

We then generalized this to obtain the equation of the surface:

$$z = 1.055 \times 10^{-5}(x^2 + y^2) - .00422. \tag{6}$$

We established that the area of maximum coverage would be the midpoint of the line connecting the two sprinklers. This was based more on intuition than anything else, but considering application rates in areas of overlap are the sums of the individual application rates per sprinkler, we felt this assumption was justified.

To calculate the maximum application rate, we determined the individual rates for each sprinkler and added the results. Given equation (5) as the coverage surface for the first sprinkler, it is a trivial matter to translate that to apply to the second sprinkler. We have

$$z = 1.055 \times 10^{-5}((x - 20)^2 + y^2) - .00422. \tag{7}$$

Assuming the origin of our coordinate system is the position of the first sprinkler, we substitute $x = 10$ and $y = 0$ into equations (4) and (5), and we find

$$z = -.003165 \text{ m per hour}$$

for each equation. The minus sign is appropriate since we are discussing depth. To obtain total application rate, we add the result from both equations, and we find

$$z = -.00633 \text{ m per hour} = -.633 \text{ cm per hour}.$$

The points of the field that obtain the maximum water application are below the 0.75 cm per hour restriction.

The total coverage of the field is illustrated below (Figure 6).

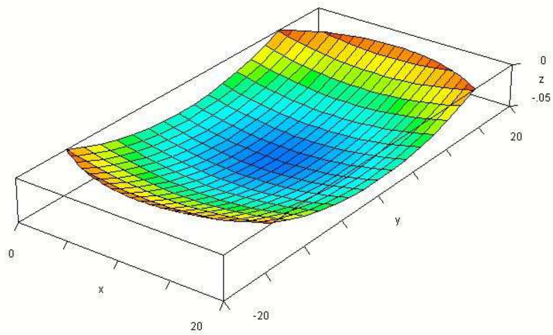

Figure 6

The sprinkler configuration constitutes a symmetric region. Using the coordinate system described above, the areas of least coverage correspond to the points where the wetting region of each sprinkler intersects with the edge of the field. The points where water coverage is minimal are (6.7712, 15) and the other three points that correspond to this relative position. The point (6.7712, 15) is where the limit of the wetting region of the sprinkler at (20, 0) intersects the edge of the field. We performed a calculation and found that this point receives 0.1363 cm per hour.

Our next objective was to determine how much water falls outside the field. Water conservation is an important issue, so we wanted to find out how much water is wasted. To do so, we assumed a one hour duration, and calculated the volume of the region bounded by the wetting radius, the edges of the field and the surface given by equation (4). In our coordinate system, this translates to the following problem statement:

Find the volume of the region bounded by

$$\begin{aligned} z &= 1.055 \times 10^{-5}(x^2 + y^2) - .00422, \\ y &= \sqrt{400 - x^2}, \\ x &= 0, \\ y &= 15, \\ z &= 0. \end{aligned}$$

The solution is to evaluate the integral

$$\int_0^{13.2288} \int_{15}^{\sqrt{400-x^2}} \left[1.055 \times 10^{-5}(x^2 + y^2) - 0.00422\right] dy\, dx, \tag{8}$$

where the upper limit of integration with respect to x is the point of intersection of the line $y = 15$ and the circle $y = \sqrt{400 - x^2}$. We evaluated this using Derive, and found that the result is -0.03459 cubic meters. This must be multiplied by four to obtain the total wasted water per hour. So every hour 0.1383 cubic meters fall outside the boundaries of the field.

In order to determine the efficiency of this configuration, we needed to figure out the total amount of water that is disbursed by the entire system. This means evaluating the integral

$$4 \int_0^{20} \int_0^{\sqrt{400-x^2}} \left[1.055 \times 10^{-5}(x^2 + y^2) - 0.00422\right] dy\, dx, \tag{9}$$

which according to Derive is −2.652 cubic meters. So the system outputs 2.652 cubic meters per hour, and 0.1383 cubic meters falls outside the field. The percentage of the water falling inside the field is approximately 95%.

5.3 Scheduling

Recall that the point of maximum coverage was .633 cm per hour, which is well under our imposed maximum application rate. The points of minimum coverage receive .1363 cm per hour. The schedule then, must be based on the points of minimal coverage. We need each part of the field to receive at least 2.00 cm of water every four days. In order to ensure this, the minimal points must be watered for an average of 3.67 hours each day over the course of four days.

5.4 Concluding Discussion

In order to minimize the time requirement for this irrigation schedule, we made use of the fact that the crops require two centimeters every four days, and developed an algorithm based on duration per section. If we water each section for 14.68 hours, then one section is watered per day. In order to minimize the amount of time the agriculturalist spends moving the pipe, we have allowed the pipe to remain stationary for this duration. The following day, the operator can move the pipe in a straight line to the next section and repeat the process.

As in the first configuration, this system has a minimum initial investment and minimum maintenance, since the operator need only worry about the main pipe and two sprinklers. Also, the time requirements on the operator are minimal, as is the labor required to move the irrigation system.

Other than the percentage of water falling outside the boundaries of the field, we were actually unable to determine any negative aspects of this model as an irrigation schedule.

If the rancher has a water pump with the irrigation system we estimate a matter of minutes to drain the system for transport, with additional minutes to relocate the pipe and refill the system. So the requirements are met, and the rancher spends perhaps less than one hour per day on irrigation maintenance and concerns.

6 Conclusion

The results for each model are summarized in the table below (Table 1):

	Estimated Daily Time Requirement	% Efficient	Maximum (cm/4 days)	Minimum (cm/4 days)
Model 1	12.18 hours	100%	3.00	0.00
Model 2	9 hours	87%	8.25	2.00
Model 3	< 1 hour	95%	9.29	2.00

Table 1.

The table is explained as follows: the estimated daily time requirement is based on our best guess as to how long it would take to relocate the irrigation system plus the actual time the operator must be on hand to observe the irrigation process. For our hasty model, we had to make the additional assumption that the irrigation system must be dismantled in order to transport it to the next station, which results in an increased time requirement.

The efficiency is determined by how much of the water output by the system actually remains in the field.

- The hasty model is the most efficient in this regard, but a large portion of the field remains dry.
- Our second model has less efficiency due to the orientation of the sprinkler heads. Since the sprinklers in the center of the pipe are operating at a narrower angle of rotation, the water is confined to a smaller area.
- The third model has a fairly good efficiency rating due to our additional assumption that the application rate varies with distance from the sprinkler.

The maximum and minimum columns are self-explanatory.

Based on our analysis, we would recommend adoption of the irrigation scheme outlined in Section 5, and shown in the diagram below. (Figure 7).

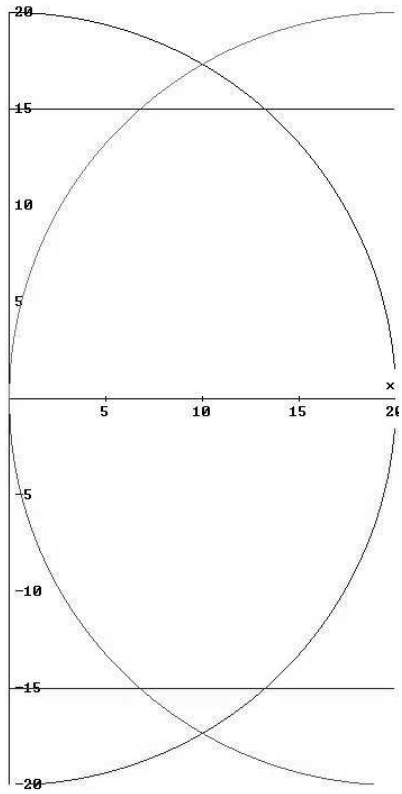

Figure 7

7 Unsatisfied Curiosities

We were somewhat hesitant to assume some of the things we did for this project. For example, we would have liked to more fully investigate the fluid dynamical aspect of sprinklers, particularly the physics of the water streaming out of the sprinkler head.

We were uncertain how insignificant air resistance is when applied to the water droplets. We attempted to model this, but the problem of projectile motion in three spatial dimensions with air resistance quickly became a problem outside of the scope of this project. We determined that this would either be a second-order nonlinear partial differential equation with 4 independent variables or a system of ordinary differential equations that share variables. If we had more time, we would have investigated this further.

We also had the idea that some sort of probability density function would help us determine more accurately the distribution of the water droplets, and would have liked to include this aspect in our third model. It was our impression that an exponential distribution would have modeled the data appropriately, but due to time limitations, this was not feasible.

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