Mathematics of the Cold War

*Author's note: This is a preprint of an article that is scheduled to appear in 2011, in the **Encyclopedia of Mathematics and Society**. This three-volume encyclopedia is intended for young adults who wish to learn about the place of mathematics in society — its meaning, its history, and its uses. It is **not** a mathematics textbook, and does **not** attempt to teach mathematics. — Loren Cobb*

The Cold War was a 45-year-long period of bitter competition between two large groups of nations. It lasted from the end of World War II in 1945 to the collapse of the Soviet Union in 1991. The two groups never came to direct combat — hence the term "cold" — but it was a war in every other way, fought with deadly ferocity in the political, economic, ideological, and technological arenas. It was also a time of unprecedented investment in new mathematical ideas, driven in part by the desire of each side to dominate the other through nuclear intimidation, economic strength, espionage, and political control.

More generally, the term "cold war" can be applied to any fight-to-the-death competition between nations in which the two sides avoid direct military combat.

In the original Cold War (1945-91), the two groups of nations divided along ideological lines. One side, the Soviet bloc, adhered to the communist political and economic philosophy of Marx and Lenin. The other side, the Western bloc, adhered to the older free-market capitalist philosophy originated by Adam Smith.

The two sides of the Cold War were essentially forced to avoid military conflict by the recent invention of the atomic bomb, because neither side wanted to risk combat that might give rise to an unstoppable military escalation. The inevitable result of such an escalation would have been worldwide nuclear war, with most large cities destroyed in an instant by nuclear warheads, followed by massive clouds of radioactive ash circling the globe and causing the death of hundreds of millions of innocent people.

Mutually Assured Destruction — the MAD doctrine

Prior to the development of the atomic bomb, there were no weapons capable of destroying the population of an entire city in a single blow. Wars were fought as purely military conflicts, without risking the life of civilization itself. All of that changed forever with the advent of nuclear weapons.

Prior to the Cold War the dominant mathematical model of warfare was a simple model of the *predator-prey* type, invented by Frederick Lanchester during World War I. Perhaps not surprisingly, given the slow, grinding progress of WWI, the Lanchester model places primary emphasis on the rates of attrition of the military forces. The side that survives this deadly attrition process wins the battle. In a nuclear-armed battle, however, it is survival rather than attrition that is the vital concern, and the Lanchester equations are irrelevant.

In the earliest years of the Cold War only the United States possessed the theory and technology to construct an atomic bomb. The presence of the bomb in the arsenal of one side but not the other made possible a strategy known as *nuclear blackmail*. The owners of the atomic bomb could threaten to use the bomb if their adversaries did not comply with their demands. For example, newly declassified documents have revealed that in 1961 Great Britain threatened China with nuclear retaliation if China were to attempt a military invasion of the British Crown Colony of Hong Kong. The United States backed up this threat, and China refrained from invading Hong Kong.

Clearly, the ownership of the secret of the atomic bomb by just one nation in 1945 had destabilized the military balance of power among the victors of World War II. Great Britain and France allied themselves with the United States and were given access to atomic secrets. The Soviet Union chose to develop its own versions of the atomic bomb, or to steal the secrets through espionage. Thus arose the great division of the Cold War, between the respective allies of the Soviet Union (known as the Warsaw Pact) on one side, and the United States (through the North Atlantic Treaty Organization, or NATO) on the other.

The Soviet Union tested its first nuclear weapon in 1949, ending its four year period of vulnerability to nuclear blackmail. The military doctrine that took its place was known as *mutually assured destruction*, or MAD. As long as each side in the Cold War could assure the other that it would be utterly destroyed in a nuclear exchange, then — so it was hoped — military conflict could be prevented. MAD did indeed prevent the two nuclear powers from directly attacking each other, but it had two unfortunate consequences: the people of both sides lived in terror of nuclear annihilation, and both superpowers engaged in so-called *proxy wars*, using much smaller nations as their proxies in localized military conflicts.

The Arms Race as a Nash Equilibrium

From its very outset, neither of the Cold War's two superpowers — the United States and the Soviet Union — believed that they could stop developing new and ever more powerful nuclear weapons. The MAD doctrine applied only as long as the forces of each side could pose a credible nuclear threat to the other. Therefore, each side worked to create new weapons as fast as possible. Throughout the 1950s and well into the 1960s, both nations tested ever more powerful nuclear weapons. This became known as the Arms Race.

In the mathematical theory of games, the military arms race brought about by the MAD doctrine is an example of a *Nash equilibrium*. In the decades-long arms race between the two Cold War competitors, each side could be seen as playing a simple non-cooperative game. Each player in this game has a choice: to construct new and more terrible nuclear weapons, or not. If either player were to chose not to develop further weapons, while the other did, then the first player would face the very real risk of eventually facing nuclear blackmail. Each player understood the other's dilemma all too well, and so both continued to develop new weapons as fast as possible.

The persistence of this behavior comes from the fact that neither player can benefit by changing strategy unilaterally. When this occurs in a game, then it is in a form of equilibrium whose existence was first proved in the general case by the mathematician John Forbes Nash in 1950.

Game theory itself was a child of the Cold War, having been created in 1944 by John von Neumann, a mathematician who also played a key role in the development of the first atomic bomb, and Oskar Morgenstern, an economist. Throughout the Cold War the theory of games was studied and elaborated, both by the military and by economists, as a means for better understanding the fundamental nature of competition, cooperation, negotiation, and war.

The fundamental irrationality of the nuclear arms race, in which each side became able to kill every single person on the planet many times over, was apparent to almost everyone. This realization did little to stop the arms race, because of the power of the Nash equilibrium to trap the players of the game into modes of behavior that, individually, they deplored.

In some critical respects, an arms race resembles a famous game known as the Game of Chicken: two cars race towards each other down a narrow road; the driver that first swerves away to avoid a crash is the loser. The key to winning a game of chicken is to act in such a way that your opponent comes to believe that you are so irrational as to be willing to die before swerving. In other words, the “rational” solution to the game is to be utterly and convincingly irrational. The same principle holds in a nuclear arms race.

The game of chicken and other apparent paradoxes of rationality within the theory of games led to the development in the 1970s of *meta-game analysis*. This and other mathematical forms of strategic analysis played an important role in the eventual winding down of the arms race with a series of strategic arms agreements between the major powers of the Cold War.

Political Competition in the Cold War

The bitter competition of the Cold War was at least as much political and economic as military, and new mathematical ideas contributed mightily to this competition. In the economic arena, the Cold War was fought between the proponents of multi-party free-market economies, on the Western side, and the proponents of single-party command economies on the Soviet side.

Both sides claimed to be democratic in the Cold War, but they used different meanings for the word. In the West the word "democracy" retained its ancient meaning, a political system in which leaders are chosen in free elections. In the Soviet system, on the other hand, "democracy" meant a "dictatorship of the proletariat", in which all political power rested in a hierarchy of labor councils, known as soviets, and the supreme soviet could dictate any aspect of public affairs. Soon after the Russian Revolution, however, the Communist Party seized control of the soviets, and after that no election in the Soviet system was free.

The intense political competition between these two systems of government led to great interest in the West in how to conduct elections in the fairest possible way. A large body of mathematical theory of elections emerged, much of it devoted to the study of election systems that come the closest to meeting a measure of fairness known as the Condorcet criterion. In an election, the *Condorcet winner* is the candidate who can beat any of the other candidates in a two-person run-off election. Many forms of preference balloting, in which voters rank the candidates, come quite close to the Condorcet criterion, but none is without problems.

Arrow's Paradox, discovered and proved by Kenneth Arrow in 1950, states that when voters have three or more choices, then no voting system can convert the ranked preferences of the voters into a community-wide ranking that meets a particular beneficial set of criteria. This Cold War mathematical discovery is the starting point of the modern theory of social choice, the foundation of the mathematical theory of political science.

Economic Competition in the Cold War

There are many forms of socialism known in economic theory, but the form practiced by the Soviet bloc of nations was particularly severe. In its purest form, Soviet socialism entailed state ownership of all means of economic production: all industrial plants, all commercial businesses, all farms, and all financial institutions. Soviet socialism was a *command economy*, meaning that the state had to tell every plant, business, and institution how much to produce, and at what price they should sell their goods and services.

In order to come up with the enormous number of production and price commands that had to be sent out every month and year, the Soviet system of government employed a vast bureaucracy. The system used by these bureaucrats was developed in the 1920s, during the early years of the Soviet Union, without the benefit of mathematics. Known as the "method of balances", this system attempted to function so that the total output of each kind of goods would match the quantity that its users were supposed to receive.

In practice, the Soviet "method of balances" functioned very much like the U.S. War Production Board during World War II, and by its counterparts in the war economies of Great Britain and Germany. The first production decisions were made with respect to the highest priority items — ships, tanks, airplanes — and those were balanced with the available amounts of strategic resources — iron, coal, electricity — and so on down to the lowest priority items. The command system was crude and error-prone, and its mistakes and imbalances were widely noticed.

In the West, the response of mathematicians to these failures of the war-time command economy was the development of the field of engineering known as *operations research*. The mathematical technique known as *linear programming* — originally a little-known Russian discovery — was successfully developed by George Dantzig and John von Neumann in 1947 to optimize production quantities under linear constraints on supplies. The Soviet economy was very slow to adopt these ideas, preferring for ideological reasons to stick with the inefficient and error-prone method of balances until very late in the Cold War.

After World War II, the nations of the West ended their wartime command economic systems and reverted back to using the free market to make price and production decisions. The Soviet Union and its allies, however, continued to rely on a large army of bureaucrats to make all economic decisions without the aid of good operational theory.

Wassily Leontief, a Russian economist working in the United States, solved one of the fundamental problems of a command economy in 1949 with his method of *input-output analysis*. This method required the creation of a very large matrix showing the contribution of each component or sector of the economy to every other component. When properly constructed, the required inputs of raw materials to the economy can be calculated from the desired outputs by matrix inversion. Once again, the Soviet Union failed to see the significance of Leontief's achievement, and did not incorporate any of his ideas into its planning system until many decades had passed.

It is one of the great ironies of the Cold War that the mathematical theories that were required to make a command economy function properly were perfected in the West, where they are now universally employed within industrial corporations — some now bigger than the entire economy of the old Soviet Union — to run their operations in the most efficient way possible. In the end the economy of the Soviet Union and its satellites was not able to keep up with the West, and in 1991 it suffered catastrophic political and economic collapse.

SEE ALSO

Atomic Bomb; Game Theory; Predator-Prey Model; World War II.

FURTHER READINGS

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