Let $b$ be an integer greater than 1. It is well known that each positive integer $N$ has a unique representation of the form

$$N = b_0 + b_1b + b_2b^2 + \cdots + b_rb^r, \quad \text{where } 0 \leq b_i \leq b - 1, \quad 0 \leq i \leq r. \quad (1)$$

The proof amounts to showing that the $b^{r+1}$ distinct expressions of the form in Eq. 1 give $b^{r+1}$ distinct numbers, and these numbers range from 0 up through $b^{r+1} - 1$.

A second useful numeration system is the following. Define the Fibonacci numbers in the slightly uncommon way: $F_0 = 1$, $F_1 = 2$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. If $d_r, d_{r-1}, \ldots, d_1, d_0$ is a binary string of length $r + 1$, put

$$N(d_r d_{r-1} \cdots d_0) = d_r F_r + d_{r-1} F_{r-1} + \cdots + d_1 F_1 + d_0 F_0. \quad (2)$$

This version of the Fibonacci sequence $\{F_n\}_{n=0}^\infty$ has the following properties.

**Theorem (Fibonacci Numbers)**

(i) For $0 \leq i < j \in \mathbb{Z}$, $F_i < F_j$, so $\{F_n\}_{n=0}^\infty$ is strictly increasing.

(ii) $F_n$ is the number of binary strings of length $n$ not containing 11 as a substring.

(iii) $F_{n+1} - 1$ is the largest integer $N$ corresponding to a binary string of length at most $n + 1$ and not having 11 as a subword.

(iv) (Zeckendorf 1939) Each positive integer may be represented in a unique way as the sum of nonconsecutive Fibonacci numbers.
This example illustrates the general theme of this talk. A general approach to numeration systems will be given and then it will be seen how to use an appropriate numeration system to count the number of strings of a certain length from a given alphabet subject to certain restrictions on the strings. We will conclude with an explanation of the strategy used to win at a game called Fibonacci Nim.