Optimizing for Multiple Portfolios

We are interested in finding multiple high-performing portfolios that are distinct from each other to balance expected returns and risk.

**Problem** - How to define the distinctness between different portfolios?

**Approach** - Query financial expert on the distinctness between a proposed portfolio $x$ and the optimal portfolio $x_{opt}$. Learn distinctness to match user preference.

Selecting Portfolio after the Search

To resolve a trading strategy based the supplemental portfolios, we formulate the multiobjective optimization problem:

$$\max_{x \in \Omega} f(x)$$
$$\max_{x \in \Omega} d(x, x_{opt})$$

Without loss of generality, we index the portfolios in non-increasing performance metric, such that

$$f(x_1) \geq f(x_2) \geq \cdots \geq f(x_n).$$

We define $x_i$ to be an $\alpha$-distinctly efficient portfolio if

$$\Pr[d(x_{opt}, x_i) > d(x_{opt}, x_j)] > \alpha, \quad 1 < j < i.$$

The free parameter $\alpha$ indicates the user's preference for distinctness, where a lower value provides more freedom for identifying the supplemental portfolios and a higher value reduces the cognitive burden of the user.

**Note** - the actual distinctness $d$ is unknown to us.

Distinctness via User Preference

**Goal** - Devise a comparison strategy which, given portfolios $w, x, y, z \in \Omega$, can estimate

$$\Pr[d(w, x) > d(y, z)],$$

where $d : \Omega \times \Omega \to \mathbb{R}_+$ is the user’s implicit sense of distinctness.

**Logistic Regression** - Equivalence of a binary classification problem with input features

$$w, x, y, z \rightarrow \left| \begin{array}{l} w = x \\ y = z \end{array} \right|$$

Convert user's rankings into $\binom{m}{2}$ pair-wise classification data points for the logistic model.

- $x_{opt}, x_1, x_{opt}, x_2$, False
- $x_{opt}, x_2, x_{opt}, x_1$, True
- ...

Interpreting Efficient Results

The solutions are visualized as a pareto frontier. As $\alpha$ decreases, more of the results are included, e.g., portfolios distinct for $\alpha = 0.7$ are a subset of those distinct for $\alpha = 0.6$.

Evaluating Portfolio Performance

We optimize for the Sharpe ratio for over aggregated industries using data from the S&P 500 during 2016. Our trading strategy consists of $x_{opt}$ supplemented equally by five distinct portfolios, and judge the empirical mean and variance over 11 trading days.

Analysis - If the user can successfully identify assets which will perform well, $x_{opt}$ can be supplemented with portfolios that decrease the variance without decreasing the mean.