A method of slowing the growth of an insect population without pesticides is to introduce a number of sterile males into the population to mate with fertile females without producing offspring. If $P$ represents the number of female insects in a population, $S$ the number of sterile males introduced each generation, and $r$ the population’s natural growth rate, then the female population is related to time $t$ by

$$t = \int \frac{P + S}{P \left[ (r - 1) P - S \right]} \, dP.$$ 

Suppose an insect population with 10,000 females grows at a rate of $r = 0.10$ and 900 sterile males are added. Evaluate the integral to give an equation relating the female population to time. (Note that the resulting equation cannot be solved explicitly for $P$.)

- Note that the population of 10,000 females is an initial condition that will be used to determine the constant of integration and should not be plugged in as $P = 10,000$ before solving the integral. The integral becomes

$$\int \frac{P + 900}{P \left( -0.90P - 900 \right)} \, dP.$$

The partial fraction decomposition of this integral has the form $\frac{A}{P} + \frac{B}{-0.90P - 900}$. Using the coverup method, we set $P = 0$ to determine $A = \frac{900}{900} = -1$ and we set $P = -1000$ to determine $B = \frac{-100}{1000} = 0.1$. Thus, the integral becomes

$$t = \int \left( -1 \left( \frac{1}{P} \right) + 0.1 \left( \frac{1}{-0.90P - 900} \right) \right) \, dP$$

$$= \int \left( -1 \left( \frac{1}{P} \right) - \frac{1}{9} \left( \frac{1}{P + 1000} \right) \right) \, dP$$

$$= - \ln |P| - \frac{1}{9} \ln |P + 1000| + c$$

Using our initial conditions of $t = 0$, $P = 10,000$, we see that $0 = - \ln 10,000 - \frac{1}{9} \ln |11,000| + c$, so that $c = \ln 10,000 - \frac{1}{9} \ln 11,000$, which is about 10.2443. Thus,

$$t = - \ln |P| - \frac{1}{9} \ln |P + 1000| + \ln 10,000 - \frac{1}{9} \ln 11,000.$$