Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 seconds. The maximum rate of air flow into the lungs is about 0.5 liters per second. This explains, in part, why the function \( f(t) = \frac{1}{2} \sin \left( \frac{2\pi t}{5} \right) \) has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time \( t \).

- Since \( f(t) \) represents the rate of air flow (that is, the derivative of the volume), if \( V(t) \) is the volume, then \( V(t) \) satisfies the conditions \( V'(t) = f(t) \) and \( V(t) \geq 0 \). Thus,
  \[
  V(t) = \int f(t) \, dt \\
  = \int \frac{1}{2} \sin \left( \frac{2\pi t}{5} \right) \, dt \\
  = \frac{5}{4\pi} \int \sin u \, du \\
  = -\frac{5}{4\pi} \cos u + c
  \]

Now the function \(-\frac{5}{4\pi} \cos u + c\) has as its minimum value \(-\frac{5}{4\pi} + c\). Since the volume cannot be negative, \( c \geq \frac{5}{4\pi} \). If equality is taken, we assume that all air is exhaled and none is retained in the lungs between breaths. Otherwise, the difference represents the residual air that remains in the lungs between breaths.

A rocket accelerates by burning fuel, decreasing its mass with time. If the rocket’s initial mass at liftoff is \( m \), fuel is consumed at rate \( r \), and the exhaust gases are ejected with constant velocity \( v_e \) relative to the rocket, a model of the velocity of the rocket at time \( t \) when \( t \) is relatively small is given by

\[
  v(t) = -gt - v_e \ln \frac{m - rt}{m},
\]

where \( g \) is the acceleration due to gravity. If \( g = 9.8 \text{ m/s}^2 \), \( m = 30,000 \text{ kg} \), \( r = 160 \frac{\text{kg}}{\text{s}} \), and \( v_e = 3000 \frac{\text{m}}{\text{s}} \), find the height of the rocket 60 seconds after liftoff.

- Since velocity is the derivative of height, in this case, we have the height given by \( \int_0^{60} v(t) \, dt \).

\[
  \int_0^{60} v(t) \, dt = \int_0^{60} \left( -gt - v_e \ln \frac{m - rt}{m} \right) \, dt \\
  = \left[ \left( -\frac{gt^2}{2} + \frac{v_e m}{r} \left( m - rt \ln \frac{m - rt}{m} - m \right) \right) \right]_0^{60} \\
  = \left( -9.8 \frac{t^2}{2} + \frac{30,000 \cdot 30,000}{160} \left( \frac{30,000 - 160t}{30,000} - \frac{30,000}{30,000} - 1 \right) \right) \left|_0^{60} \right. \\
  = 14,844
  
\]