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Biography Paper – Georg Cantor

Few mathematicians are house-hold names; perhaps only Newton and Euclid would qualify. But there is a second tier of mathematicians, those whose names might not be familiar, but whose discoveries are part of everyday math. Examples here are Napier with logarithms, Cauchy with limits and Georg Cantor (1845 – 1918) with sets.

In fact, those who superficially familiar with Georg Cantor probably have two impressions of the man: First, as a consequence of thinking about sets, Cantor developed a theory of the actual infinite. And second, that Cantor was a troubled genius, crippled by Freudian conflict and mental illness.

The first impression is fundamentally true. Cantor almost single-handedly overturned the Aristotle's concept of the potential infinite by developing the concept of transfinite numbers. And, even though Bolzano and Frege made significant contributions, "*Set theory ... is the creation of one person, Georg Cantor.*" [4]

The second impression is mostly false. Cantor certainly did suffer from mental illness later in his life, but the other emotional baggage assigned to him is mostly due his early biographers, particularly the infamous E.T. Bell in *Men Of Mathematics* [7]. In the racially charged atmosphere of 1930's Europe, the sensational story mathematician who turned the idea of infinity on its head and went crazy in the process, probably make for good reading. The drama of the controversy over Cantor's ideas only added spice.

Fortunately, modern scholars have corrected the errors and biases in older biographies. [9,12] The modern view paints Georg Cantor in a much more favorable light. In fact, the most scholarly work on Cantor even suggests that his bipolar disorder may have contributed to his success as a mathematician. [9,10]

There is no doubt that Cantor was a somewhat quirky guy. Throughout his life he maintained intense, serial friendships. The most notable examples are Dedekind and later Mittag-Leffler. Cantor would carry on intense correspondences with his 'friend' for a period of years, then have an absolute falling out. In a way this is fortunate. His literary estate is in the care of the Akademie der Wissenschaften in Gottingen. Although most family letters have been lost, copies of Cantor's letters, as well as copies kept by others, memoirs, and reminiscences of people who knew Cantor, have allowed modern biographers to piece together a more accurate view of Georg Cantor's life. [10]

Cantor was also very philosophical about his mathematics. Later in his career he defended his work from theological and philosophical, as well as mathematical, points of view. Perhaps this lifelong philosophical bent comes from his family history.

Cantor's father, Georg Waldemar Cantor was probably born in Copenhagen around 1809. According to family lore, the family lost everything in the English bombardment of 1807, and moved to St Petersburg, Russia. Georg W. was a devout Lutheran, and had a deep love of culture and the arts. [10] Georg Waldemar Cantor was a successful merchant, working as a wholesaling agent in St Petersburg, then later as a broker in the St Petersburg Stock Exchange.

In 1842, Georg W. married Maria Anna Böhm. Maria had been born in St. Petersburg, and was baptized a Roman Catholic. The Böhm family was very artistic and musical. There were several noted classical violinists in the family.

Georg Ferdinand Ludwig Philipp Cantor was born in 1845. His family lived in the Western merchant colony in Saint Petersburg, Russia, and Georg lived there until he was eleven. Georg was the eldest of six children. Religion was a big part of Cantor family life, and all the children were brought up as Lutherans. All the children also showed artistic talent.

In 1856, when Cantor was 11, Georg W. became ill and the family moved to Wiesbaden, Germany. Georg attended the Gymnasium (or college prep school) there. Then he studied at Realschule in Darmstadt (near Frankfurt), Germany where he lived as a boarder. He graduated in 1860 with “an outstanding report,” which mentioned in particular his exceptional skills in mathematics, particularly trigonometry.

Cantor attended the Höhere Gewerbeschule in Darmstadt from 1860 to 1862. Georg W. wanted his son to be an engineer, but Georg wanted to be a mathematician. Bell made a great deal about this conflict; it was probably the basis of his half-baked Freudian theories about Cantor. The fact is, based on family letters, that Georg W. was a sensitive man who cared about his children’s wishes. [10]

So, with the consent of his father, Georg entered he entered the Polytechnic of Zurich in 1862. His studies at Zurich, however, were cut short by the death of his father in June 1863. Cantor shifted his studies to the prestigious University of Berlin. While there, some of his lecturers were Kronecker, Karl Weierstrass, and Ernst Kummer.

Cantor was not a recluse at Berlin. Georg became involved with the Mathematical Society, and was its president from 1864-65. Georg spent the summer of 1866 at the University of Göttingen, another prestigious center of European mathematics. In 1867, he received his PhD from the U. of Berlin for a thesis on number theory, *De aequationibus secundi gradus indeterminatis*.

Cantor published a number of good, but not great, articles on number theory between 1867 and 1871. [3] In 1868, he joined the Schellbach Seminar for mathematics teachers. And in 1869, he was appointed to the University of Halle in Halle, Germany. He was awarded the requisite habilitation in 1869 for his thesis on number theory. The habilitation, known as *Privatdozent* in German, is the highest academic qualification a person could achieve in Europe. Halle was not as prestigious as either Berlin or Göttingen; nevertheless Cantor spent his career there.

At Halle, Cantor was influenced by Eduard Heine (1821—1881), one of his senior colleagues. Heine challenged Cantor to prove the open problem on the uniqueness of representation of a function as a trigonometric series, the Fourier series. So Cantor moved from number theory to analysis. [3]

Cantor solved the problem proving uniqueness of the representation by April 1870. In the proof, Cantor uses the requirement that zero be represented by a convergent trigonometric series at each point of the domain. In 1871, he proved that the theorem remains valid even if there are a finite number of points in the domain at which either the convergence of the series or the representation of zero does not hold. [3]

The year 1892 was big for Georg Cantor. In 1872, he wrote a paper that extends the idea to an infinite number of points, provided they are distributed in a certain way (based on an idea due to Hermann Hankel -- 1839-1873). In the process of working on the Fourier series, Cantor began considering infinite sets with particular properties. Other consequences of this work were Cantor's first ideas on set theory, and also ideas about irrational numbers.

It is at this time the Kronecker makes another appearance in Cantor's story. There is evidence that his former lecturer was on good terms with Cantor during his first years at Halle.

However, as Cantor's ideas about infinite sets developed, Kronecker became his most prominent critic.

Also in 1872, Cantor made a trip to Switzerland. He met Richard Dedekind and he formed one of his great serial friendships. Dedekind had also been working on irrational numbers and had just published *Continuity and irrational numbers*. Many letters between the two during the years 1873-1879 are preserved. These letters contain relatively little mathematics, but they give us insight into Cantor's state of mind during this time. Certainly, Dedekind's deep abstract logical way of thinking was a major influence on Cantor as his ideas developed.

In 1874, Cantor married Vally Guttman. (They had six children, the last born in 1886). He spent much of his honeymoon in Switzerland, talking with Dedekind. Also, in 1884, Cantor published an article in Crelle's Journal which marks the birth of set theory. Here Cantor considers at least two different kinds of infinity. Before this, orders of infinity did not exist but all. Since Aristotle, infinite collections were considered 'the same size'. Needless to say, Cantor's idea was controversial.

Among other things, in his 1874 paper Cantor shows that the algebraic numbers are 'countable' in that they can be put in a one-to-one correspondence with the natural numbers. (*Note that Cantor does not use the word 'countable' at this time. He introduces that word in 1883. [4]*) On the other hand, he also showed that the real numbers are not 'countable.' A consequence of this is that there must be many more transcendental numbers than algebraic numbers.

Cantor submitted a follow-up paper to Crelle's Journal in 1878. Kronecker was on the editorial staff of Crelle's Journal, and he did not accept the revolutionary new ideas in Cantor's paper. Kronecker was the prototype example of a Finitist. He believed that a mathematical object does not exist unless it can be constructed from the natural numbers. Cantor was tempted to

withdraw his paper, but Dedekind persuaded him to submit it. Weierstrass also supported publication. The paper was published but Cantor never submitted any further work to Crelle's Journal.

In this paper, Cantor introduces the idea of equivalence of sets and says two sets are equivalent or have the same *power* if they can be put in 1-1 correspondence. Cantor took the word 'power' from Steiner. He also proves that the rational numbers have the smallest infinite power and also shows that \mathbb{R}^n has the same power as \mathbb{R} . [4]

While Cantor was doing this ground-breaking work, some political intrigue entered his life. In 1881, Cantor's Halle colleague Eduard Heine died, creating a vacant chair at the university. Halle accepted Cantor's suggestion that it be offered to Dedekind, Heinrich Weber and Franz Mertens, in that order, but each declined the chair after being offered it. (Friedrich Wangerin was eventually appointed.) Cantor became angry with Dedekind for not accepting the position, and cut-off his correspondence with him.

In 1883, Cantor wrote another paper discussing well-ordered sets, *Grundlagen einer allgemeinen Mannigfaltigkeitslehre*. Here he introduces the idea of Ordinal numbers. He also talks about the multiplication and addition of transfinite numbers. By this time, Cantor must have been aware of the controversy surrounding his work. He spends quite a bit of space in the 1883 paper defending his ideas, both mathematically and philosophically. [9]

By 1884, Cantor had moved to Mittag-Leffler as his confidant. He also had the first of his documented fits of depression and was hospitalized for a time. During this period, Cantor seemed to lose confidence in his mathematical work. He attempted to become a lecturer in philosophy at Halle. (Cantor had always had a philosophical perspective on his math.).

This period of depression was short-lived, and by 1885 Cantor returned to form. In 1885 Cantor continued to develop his theory of cardinal and ordinal numbers. In 1895—1896, Cantor published a final treatise on sets theory. The *Beitrage zur Begrundung der transfiniten Mengenlehre*, contains an introduction that looks like a modern textbook, defining the terms set, subset, and so on. [3]. In this paper, unlike the Grundlagen, Cantor kept the philosophical side of his mathematics under the surface. Perhaps he felt confident enough in his theory to let it stand on its own terms, without need for further philosophical and theological explanation. [9]

As an aside, at about this time, Cantor became obsessed with the idea that Francis Bacon wrote the plays attributed to William Shakespeare. He published two pamphlets on this subject in 1896 – 1897. [1]

Also in 1897, the first published paradox in set theory was published by Cesare Burali-Forti. It revolves round the set of all ordinal numbers. The ordinal number of the set of all ordinals must be an ordinal and this leads to a contradiction. There is evidence that Cantor discovered this paradox himself in 1885 and wrote to Hilbert about it in 1886. [3]

By 1897, Cantor's work began to gain supporters. The first International Congress of Mathematicians was held in Zurich and at the conference Cantor's work was held in the highest esteem. His work was praised by many mathematicians, including Hurwitz and Hadamard. Also, at the Congress, Cantor saw Dedekind and they renewed their friendship. [3]

At this point, Cantor's mental health truly begins to decline. He was hospitalized again in 1899, 1904, and on and off every couple years until his death. While he was giving one of his Bacon lectures in 1899, Cantor learned that his youngest son had died suddenly. This tragedy seemed to drain Cantor of his passion for mathematics for the rest of his life.

However, in his declining years, Cantor continued to receive accolades. He attended the International Congress of Mathematicians at Heidelberg in 1904, Cantor was one of the distinguished foreign scholars invited to attend the 500th anniversary of the founding of the University of St. Andrews in Scotland in 1911, and was awarded an honorary doctorate, by St. Andrews in 1911, but he was too ill to attend. Cantor retired from Halle in 1913, and suffered poverty and malnutrition during WWI. In 1915, a public celebration of Cantor's 70th birthday was scheduled, but then cancelled due to the war.

In 1915, Philip Jourdain published the first English translation of Georg Cantor's two important papers on cardinal and ordinal numbers.[8] He added a long introduction which takes 85 pages of the 200 page book. Jourdain apparently felt that Cantor's ideas needed a lengthy introduction and justification to put them in context.

Finally, Georg Cantor died on January 6, 1918 in the sanatorium where he spent the final year of his life.

After 1897, more set theory paradoxes began to appear. In 1899 Cantor himself discovered another paradox which arises from the set of all sets. The paradox was: what is the cardinal number of the set of all sets? It must be the greatest possible cardinal number, yet set theory said that the cardinal number of the set of all subsets of a set always has a greater cardinal than the set itself. [3]

The 'ultimate' set paradox was found by Bertrand Russell in 1902 (and found independently by Zermelo). The Russell paradox revolved around sets that do not contain themselves as members. In the classic example, suppose a barber shaves men if and only if they do not shave themselves. The question is, should the barber shave himself or not? [2]

These paradoxes are legacies of Cantor's work. Ultimately, they lead to the development of axiomatic set theory and on to Gödel's incompleteness theorems. Cantor was also influential in the Formal work of David Hilbert. In fact, Cantor's ideas come into play in the still ongoing debate over the various schools of mathematical thought – Logicism (Russell), Formalism (Hilbert) , Intuitionism (Brouwer) , and Finitism (the realm of Kronecker).

But Cantor's greatest legacy is set theory itself. Cantor's idea of sets and a hierarchy of infinite sets has become the foundation of modern mathematics. Educators even experiment with teaching sets to elementary school children. This is quite an accomplishment for a man who historians wrote off as 'crazy' 50 years ago.

From a modern perspective, Cantor was anything but a bundle of Freudian conflicts. Despite his mental problems (*which today we would ascribe to 'bipolar disorder,' and probably treat successfully with one of the medications advertised on television*), Cantor accomplished a great deal. He not only had a long and productive career, producing a string of ground breaking papers, but he defended his work amongst sometimes blistering and personal criticism. Poincare called the theory of transfinite numbers a "disease" from which he was certain mathematics would someday recover from. Kronecker went so far as to call Cantor a "scientific charlatan," and a "corrupter of youth." [10] Early on, Cantor himself had trouble accepting the consequences of his work, and his own doubts gave him insight into possible objections. [3]

Joseph Dauben is one of the most prominent modern Cantor biographers. He has put forth a theory that Cantor's bipolar (manic-depressive) disorder helped, rather than hindered, his mathematics. Dauben suggests that the manic phases of Cantor's disease contributed to the energy and single-mindedness with which he defended his theory.

References

- [1] Georg Cantor. Web page, Wikipedia, 2008. Available at http://en.wikipedia.org/wiki/Georg_Cantor.
- [2] Russell's Paradox. Web page, Wikipedia, 2008. Available at http://en.wikipedia.org/wiki/Russell's_paradox.
- [3] MacTutor Archives. Georg Ferdinand Ludwig Philipp Cantor. Web page, University of St. Andrews, Scotland, 2008. Available at <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Cantor.html>.
- [4] MacTutor Archives. A history of set theory. Web page, University of St. Andrews, Scotland, 2008. Available at http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Beginnings_of_set_theory.html.
- [5] MacTutor Archives. Julius Wilhelm Richard Dedekind. Web page, University of St. Andrews, Scotland, 2008. Available at <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Dedekind.html>.
- [6] MacTutor Archives. Leopold Kronecker. Web page, University of St. Andrews, Scotland, 2008. Available at <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Kronecker.html>.
- [7] E. T. Bell. *Men of Mathematics*. Simon and Schuster, New York, NY, 1937.
- [8] G. Cantor. *Contributions to the Funding of the Theory of Transfinite Numbers*. Dover Publications, Inc., New York, NY, 1915.
- [9] J. W. Dauben. *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Princeton University Press, Princeton, NY, 1979.
- [10] J. W. Dauben. Georg cantor and the battle for transfinite set theory. *Journal of the ACMS*, 2004.
- [11] S. Hollingdale. *Makers of Mathematics*. Penguin Books, New York, NY, 1994.
- [12] V. J. Katz. *A History of Mathematics: An Introduction*. Addison Wesley Longman, Inc., Reading, MA, 2 edition, 198.