Remember that only the starred ★ problems are going to be graded. The rest of the problems are just good practice.

**Chapter 5.1**

1. From an ordinary deck of 52 cards, cards are drawn at random and with replacement. What is the probability that, of the first 8 cards drawn, 4 are spades?

4. ★ In a state where license plates contain six digits (randomly selected between 0-9), what is the probability that the license has two 9’s?

6. A manufacturer of nails claims only 3% are defective. A random sample of 24 nails is selected, of which 2 are defective. How do you feel about his 3% claim based on these results?

11. Let $X \sim \text{Binom}(n, p)$ with probability mass function $p(x) = P(X = x)$. Prove that if $(n + 1)p$ is an integer, then $p(x)$ is maximized at two different points. Find these two points.

17. ★ What is the probability that at least two of the six members of a family are not born in the fall?

24. In a game, a player bets on an integer 1-6. Three dice are tossed, and if one, two or three land on the player’s number, then the player wins one, two or three times their bet, plus their original wager back. If none of the dice land on the player’s number, the player loses their bet. Let $X$ be the net gain of the player. Find the probability mass function of $X$ and the expected value of $X$.

33. ★ An urn contains $n$ balls whose colors are equally probably: e.g., the probability that all of the balls are red is $(1/2)^n$. If in drawing $k$ balls from the urn, with replacement, no red balls appear, what is the probability that the urn contains no red balls?

**Chapter 5.2**

1. Jim buys 60 lottery tickets every week. If only 5% of the lottery tickets win, what is the probability that he wins next week?

5. ★ On a random day, the number of vacant rooms of a big hotel in New York City is 35, on average. What is the probability that next Saturday this hotel has at least 30 vacant rooms?

7. ★ Suppose that $X$ is a Poisson random variable with $P(X = 1) = P(X = 3)$. Find $P(X = 5)$.

12. Suppose that, in Japan, earthquakes occur at a Poisson rate of 3 per week. What is the probability that the next earthquake occurs after two weeks?

21. Suppose that in Maryland, on a certain day, $N$ lottery tickets are sold and $M$ win. To have a probability of at least $\alpha$ of winning on that
day, approximately how many tickets should be purchased?

**Chapter 5.3**

1. Define the sample space so that, from a box containing two broken and five working items, three items are drawn at random and without replacement.

4. The probability is $p$ that Marty hits his target, and $q$ that Alvie hits his target. If both hit their tarts, they stop; otherwise they both shoot again.
   a) What is the probability that they stop after each has fired $r$ times?  
   b) What is the expected value of the number of times each shoots?

5. Suppose that 20% of a group of people have hazel eyes. What is the probability that the eighth passenger boarding a plane is the third to have hazel eyes?

8. The probability is $p$ that a randomly chosen light bulb is defective. We test a bulb and if it works we stop; otherwise we throw that bulb away and test another. What is the probability that at least $n$ bulbs are required?

12. On average, how many games of bridge are necessary before a player is dealt three aces? A bridge hand is 13 random cards from a deck of 52.

20. A fair coin is flipped repeatedly. What is the probability that the fifth tail occurs before the tenth head?

24. 1200 eggs, of which 200 are rotten, are distributed randomly in 100 cartons of one dozen each. How many carts should we expect the chef of the restaurant to open before finding one without rotten eggs?