Question 1

Consider the ODE
\[ y''' - y' = x. \]

Question 1.a

Study the order of this ODE. How many unknown coefficients are expected in the solution \( y(x) \)?

Answer 1.a

The order of this ODE is 3 (the highest derivative is \( y''' \)) so we expect 3 unknown coefficients in the solution.

Question 1.b

Determine if \( y(x) = Ce^x + x^2/2 \) is a solution for any value \( C \in \mathbb{R} \).

Answer 1.b

We simply need to plug this in, which requires \( y'(x) = Ce^x + x \) and \( y'''(x) = Ce^x \). This gives us
\[ y''' - y' = Ce^x - (Ce^x + x) = Ce^x - Ce^x - x = -x \neq x, \]
thus \( y(x) = Ce^x - x^2/2 \) is not a solution, regardless of the value of \( C \).

Question 1.c

Determine if \( y(x) = D \) is a solution for any \( D \in \mathbb{R} \).

Answer 1.c

Again, we simply plug this in using \( y'(x) = 0 \) and \( y'''(x) = 0 \) which gives
\[ y''' - y' = 0 - 0 = 0 \neq x, \]
thus \( y(x) = D \) is not a solution.
Question 2

Solve the separable initial value problem

\[ y' = y \cos x + y, \quad y(0) = 1. \]

Answer 2

We need to convert this to the standard form \( f(y)y' + g(x) = 0: \)

\[ \frac{1}{y} - \cos x - 1 = 0; \]

therefore, \( f(y) = 1/y \) and \( g(x) = -\cos x - 1. \) Antidifferentiating gives us

\[ F(y) = \log y, \quad G(x) = \sin x - x. \]

In class we showed that the solution is \( F(y) + G(x) = C, \) so we have the implicit ODE solution

\[ \log y - \sin x - x = C. \]

Using the initial condition we can solve for \( C: \)

\[ \log 1 - \sin 0 - 0 = C \quad \Rightarrow \quad C = 0, \]

which gives the implicit IVP solution as

\[ \log y - \sin x - x = 0. \]

We can solve for \( y \) explicitly to get

\[ y(x) = e^{\sin x + x}. \]